

# Lecture # 24 – Cost-Benefit Analysis

## I. Steps to Cost-Benefit Analysis (continued)

### 4. Discounting

- The costs and benefits will occur at different times. To compare them fairly, it is important to discount costs and benefits that occur in the future.
  - The idea is to compare a flow of benefits and costs into a single value.
- The present value of a future amount of money is the maximum amount you would be willing to pay today for the right to receive that money in the future.
  - Present value accounts for the opportunity cost of not investing the money elsewhere.
  - Example:
    - You have \$100 now
    - If you put it in the bank, you will get 5% interest
    - Next year, that money is worth  $(1 + 0.05) \times 100 = \$105$
    - After two years, it is worth  $(1.05)(1.05)(100) = (1.05)^2(100) = \$110.25$
  - General rule:
    - $PV = \text{present value (the principal that you invest)}$   $r = \text{interest rate}$
    - $\text{Future Value} = FV = PV(1 + r)^t$
  - As a result, you wouldn't give up \$100 now for \$100 next year, because you could invest the money and get \$105 next year.
    - The present value of \$100 next year is the most you would give up today to get \$100 next year
    - $FV = \$100 = PV(1.05) = \$100$
    - $PV = 100/1.05 = \$95.24$
  - General rule
    - $PV = FV/(1 + r)^t$
  - For a stream of payments:
    - $PV = FV + FV/(1+r) + FV/(1+r)^2 + \dots + FV/(1+r)^t$
  - For payments forever:
    - $PV = FV/r$

- What about inflation?
  - When we are discounting costs & benefits, we must remember that inflation affects the interpretation.
  - The key is to be consistent.
    - If all the dollar values are real dollars – already adjusted for inflation – use a real rate of interest.
    - If all the dollar values are nominal dollars – we need to adjust for inflation. We use a nominal rate of interest.
  - Recall our example from before. Suppose inflation is 3%. Then, \$105 next year is only worth  $105/1.03 = \$101.94$  today.
    - The present value now is  $100(1 + r) = \$101.94$
    - This implies a rate of return of approximately 2%.
    - *Inflation lowers the real return on our investment.*
  - For those interested in the derivation of the real rate of return, define the following terms:
    - $r$  = real rate of interest
    - $\gamma$  = rate of inflation
    - $m$  = nominal rate of interest
    - the nominal return  $(1 + m) = (1 + r)(1 + \gamma) = 1 + r + r\gamma + \gamma$ 
      - $\Rightarrow m = r + r\gamma + \gamma$
      - or  $r = (m - \gamma)/(1 + \gamma)$
    - In practice,  $r\gamma$  is very small, so a useful approximation is:
      - $m = r + \gamma$  or
      - $r = m - \gamma$
      - E.g. nominal rate = 5%, inflation = 3%  $\Rightarrow$  real rate = 2%
    - Key point: if nominal prices are used, discount using a nominal rate of interest, so that inflation is accounted for. If real prices are used, inflation has already considered. A nominal rate would double count inflation. Therefore, use a real rate of interest.
      - In general, the real rate of interest will be around 3-5%. In the next class we will discuss more carefully how to select the discount rate.
- What about capital?
  - Since we are discounting, the cost of capital should be included in the year the capital is purchased.
  - Depreciation is not necessary. The full cost is already considered when the cost of purchasing the capital is recorded. Adding depreciation would lead to double counting.
    - However, if the capital is not completely used, the salvage value should be added as a benefit at the end of the program.

## 5. Interpreting the Results

- Once our calculations are complete, how do we interpret the final results? There are several alternative criteria.

1. Net present value (NPV) – the present value of benefits minus the present value of costs

- If NPV is positive, the project is worth doing.
- If several projects are under consideration, choose the one with the highest NPV.
- Advantage: not sensitive to whether something is recorded as a cost or a benefit.
- Example:
  - Consider two projects: both have costs in first year, followed by annual benefits received forever
  - Discount rate = 5%

A			B		
Year	Benefit	Cost	Year	Benefit	Cost
0	\$0	\$1000	0	\$0	\$100
1+	\$100	\$0	1+	\$20	\$0

$$NPV_A = 0 - \$1,000 + \frac{\$100 - 0}{0.05} = -\$1,000 + \$2,000 = \$1,000$$

$$NPV_B = 0 - \$100 + \frac{\$20 - 0}{0.05} = -\$100 + \$400 = \$300$$

- Project A has the higher NPV

2. Internal Rate of Return – the discount rate that would make a project's net present value equal zero.

- A project is worthwhile if the IRR is greater than the opportunity cost of funds for the community (that is, the appropriate discount rate).
- Problem: Doesn't account for the size of the project
- Continue with the example from above

$$IRR_A: -\$1,000 + \$100/r = 0 \qquad IRR_B: -\$100 + \$20/r = 0$$

$$\$100/r = \$1,000$$

$$\$20/r = \$100$$

$$\$100/\$1,000 = r$$

$$\$20/\$100 = r$$

$$r = 0.1$$

$$r = 0.2$$

- Although project A has a higher net present value, project B has a higher internal rate of return.

3. Benefit –cost ratio – the ratio of the present value of a stream of benefits to the present value of the stream of costs.

- A project is acceptable if BC ratio > 1.
- Problems
  - Does not take the scale of the project into account.
    - Continue with the example from above
      - Project A: \$2,000/\$1,000 = 2
      - Project B: \$400/\$100 = 4
    - Although project A has a higher net present value, project B has a higher benefit-cost ratio.
  - More importantly, benefit-cost ratios are sensitive to whether items are recorded as costs or benefits.
    - Example:

A			B		
Year	Benefit	Cost	Year	Benefit	Cost
0	\$0	\$35,000	0	\$40,000	\$0
1	\$40,000	\$0	1	\$0	\$40,000

$$NPV_A = 0 - \$35,000 + \frac{\$40,000 - 0}{1.05} = -\$35,000 + \$38,095.24 = \$3,095.24$$

$$NPV_B = \$40,000 - 0 + \frac{\$0 - \$40,000}{1.05} = \$40,000 - \$38,095.24 = \$1,904.76$$

- Project A has the higher NPV
- Project A also has the highest benefit-cost ratio
  - A: B = \$40,000/1.05 = \$38,095.24
  - C = \$35,000
  - B/C = 1.088
  - B: B = \$40,000
  - C = \$40,000/1.05 = \$38,095.24
  - B/C = 1.05
- Now redefine the costs and benefits. Suppose we consider part of project B's benefit a negative cost instead (e.g. a cost savings):

A			B		
Year	Benefit	Cost	Year	Benefit	Cost
0	\$0	\$35,000	0	\$10,000	-\$30,000
1	\$40,000	\$0	1	\$0	\$40,000

- The NPV remains the same. There is still a \$40,000 net benefit in year 0.
- Recalculate the benefit-cost ratio for project B
  - B: B = \$10,000
  - C = -30,000 + \$40,000/1.05 = \$8,095.24
  - B/C = 10,000/8,095.24 = 1.24
- Now project B looks better. **Simply changing how we define benefits and costs changes the interpretation of the benefit-cost ratio.**

- As a result of all of this, NPV is the best method to use.

- The interpretation of net present value (NPV) depends on the situation. Key questions:
  1. Are there several projects being considered?
    - If not, the project is acceptable if the NPV is positive.
    - If there are several alternative projects, choose the one with the highest NPV.
  2. Does the community budget limit the size of the project?
    - If so, choose the combination of projects that maximize net benefits, given the community's budget constraints.
    - Example:

	<i>Benefits</i>	<i>Costs</i>	<i>Net Benefits</i>	<i>Benefits/Costs</i>
<i>Project A</i>	10	1	9	10
<i>Project B</i>	22	10	12	2.2
<i>Project C</i>	8	4	4	2
<i>Project D</i>	4	2	2	2
<i>Project E</i>	8	10	-2	0.8

- In this case, if there is no constraint, choose A, B, C & D
  - If there is a constraint, find the best combination
    - For example, consider a \$10 budget
    - Choosing A, C, & D give a net benefit of \$15, whereas B alone only gives net benefit of \$12.
  - A note on the benefit-cost ratio
    - This is an example of how the benefit/cost ratio doesn't select the project with the highest NPV.
      - NPV is highest for B, but the benefit/cost ratio is highest for A.
      - However, when we face a budget constraint, the benefit/cost ratio helps us identify the project that has the most "bang for the buck."
3. Can the scale of the project be varied?
    - If so, choose the size where marginal benefits = marginal costs.