Lecture # 24 – Cost-Benefit Analysis

I. Steps to Cost-Benefit Analysis (continued)

4. Discounting

- The costs and benefits will occur at different times. To compare them fairly, it is important to discount costs and benefits that occur in the future.
    - The idea is to compare a flow of benefits and costs into a single value.
- The present value of a future amount of money is the maximum amount you would be willing to pay today for the right to receive that money in the future.
    - Present value accounts for the opportunity cost of not investing the money elsewhere.
    - Example:
        - You have \$100 now
        - If you put it in the bank, you will get 5% interest
        - Next year, that money is worth (1 + 0.05)x100 = \$105
        - After two years, it is worth $(1.05)(1.05)(100) = (1.05)^2(100) =$ \$110.25
    - General rule:
        - PV = present value (the principal that you invest) r = interest rate
        - Future Value = $FV = PV(1 + r)^t$
    - As a result, you wouldn't give up \$100 now for \$100 next year, because you could invest the money and get \$105 next year.
        - The present value of \$100 next year is the most you would give up today to get \$100 next year
        - FV = \$100 = PV(1.05) = \$100
        - PV = 100/1.05 = \$95.24
    - General rule
        - $PV = FV/(1 + r)^t$
    - For a stream of payments:
        - $PV = FV + FV/(1+r) + FV/(1+r)^2 + \ldots + FV/(1+r)^t$
    - For payments forever:
        - $PV = FV/r$

- What about inflation?
  - When we are discounting costs & benefits, we must remember that inflation affects the interpretation.
  - The key is to be consistent.
    - If all the dollar values are real dollars – already adjusted for inflation – use a real rate of interest.
    - If all the dollar values are nominal dollars – we need to adjust for inflation. We use a nominal rate of interest.
  - Recall our example from before. Suppose inflation is 3%. Then, \$105 next year is only worth 105/1.03 = \$101.94 today.
    - The present value now is 100(1 + r) = \$101.94
    - This implies a rate of return of approximately 2%.
    - *Inflation lowers the real return on our investment.*
  - For those interested in the derivation of the real rate of return, define the following terms:
    - r = real rate of interest
    - γ = rate of inflation
    - m = nominal rate of interest
    - the nominal return $(1 + m) = (1 + r)(1 + \gamma) = 1 + r + r\gamma + \gamma$
      - $\Rightarrow m = r + r\gamma + \gamma$
      - or $r = (m - \gamma)/(1 + \gamma)$
    - In practice, $r\gamma$ is very small, so a useful approximation is:
      - $m = r + \gamma$ or
      - $r = m - \gamma$
      - E.g. nominal rate = 5%, inflation = 3% => real rate = 2%
    - Key point: if nominal prices are used, discount using a nominal rate of interest, so that inflation is accounted for. If real prices are used, inflation has already considered. A nominal rate would double count inflation. Therefore, use a real rate of interest.
      - In general, the real rate of interest will be around 3-5%. In the next class we will discuss more carefully how to select the discount rate.
- What about capital?
  - Since we are discounting, the cost of capital should be included in the year the capital is purchased.
  - Depreciation is not necessary. The full cost is already considered when the cost of purchasing the capital is recorded. Adding depreciation would lead to double counting.
    - However, if the capital is not completely used, the salvage value should be added as a benefit at the end of the program.

5. Interpreting the Results

- Once our calculations are complete, how do we interpret the final results? There are several alternative criteria.
  1. Net present value (NPV) – the present value of benefits minus the present value of costs
     - If NPV is positive, the project is worth doing.
     - If several projects are under consideration, choose the one with the highest NPV.
     - Advantage: not sensitive to whether something is recorded as a cost or a benefit.
     - Example:
       - Consider two projects: both have costs in first year, followed by annual benefits received forever
       - Discount rate = 5%

	A			B	
Year	**Benefit**	**Cost**	**Year**	**Benefit**	**Cost**
0	\$0	\$1000	0	\$0	\$100
1+	\$100	\$0	1+	\$20	\$0

$$NPV_A = 0 - \$1,000 + \frac{\$100 - 0}{0.05} = -\$1,000 + \$2,000 = \$1,000$$

$$NPV_B = 0 - \$100 + \frac{\$20 - 0}{0.05} = -\$100 + \$400 = \$300$$

       - Project A has the higher NPV

  2. Internal Rate of Return – the discount rate that would make a project's net present value equal zero.
     - A project is worthwhile if the IRR is greater than the opportunity cost of funds for the community (that is, the appropriate discount rate).
     - Problem: Doesn't account for the size of the project
     - Continue with the example from above

IRR_A: -\$1,000 + \$100/r = 0	IRR_B: -\$100 + \$20/r = 0
\$100/r = \$1,000	\$20/r = \$100
\$100/\$1,000 = r	\$20/\$100 = r
r = 0.1	r = 0.2

     - Although project A has a higher net present value, project B has a higher internal rate of return.

3. <u>Benefit –cost ratio</u> – the ratio of the present value of a stream of benefits to the present value of the stream of costs.
    - A project is acceptable if BC ratio > 1.
    - Problems
        - Does not take the scale of the project into account.
            - Continue with the example from above

              Project A: $2,000/$1,000 = 2

              Project B: $400/$100 = 4
            - Although project A has a higher net present value, project B has a higher benefit-cost ratio.
        - More importantly, benefit-cost ratios are sensitive to whether items are recorded as costs or benefits.
            - Example:

A			B		
Year	**Benefit**	**Cost**	**Year**	**Benefit**	**Cost**
0	$0	$35,000	0	$40,000	$0
1	$40,000	$0	1	$0	$40,000

$$NPV_A = 0 - \$35,000 + \frac{\$40,000 - 0}{1.05} = -\$35,000 + \$38,095.24 = \$3,095.24$$

$$NPV_B = \$40.000 - 0 + \frac{\$0 - \$40,000}{1.05} = \$40,000 - \$38,095.24 = \$1,904.76$$

- Project A has the higher NPV
- Project A also has the highest benefit-cost ratio

  A: B = $40,000/1.05 = $38,095.24

  C = $35,000

  B/C = 1.088

  B: B = $40,000

  C = $40,000/1.05 = $38,095.24

  B/C = 1.05
- Now redefine the costs and benefits. Suppose we consider part of project B's benefit a negative cost instead (e.g. a cost savings):

A			B		
Year	**Benefit**	**Cost**	**Year**	**Benefit**	**Cost**
0	$0	$35,000	0	$10,000	-$30,000
1	$40,000	$0	1	$0	$40,000

- The NPV remains the same. There is still a $40,000 net benefit in year 0.
- Recalculate the benefit-cost ratio for project B

  B: B = $10,000

  C = -30,000 + $40,000/1.05 = $8,095.24

  B/C = 10,000/8,095.24 = 1.24
- Now project B looks better. ***Simply changing how we define benefits and costs changes the interpretation of the benefit-cost ratio.***

- As a result of all of this, NPV is the best method to use.

- The interpretation of net present value (NPV) depends on the situation. Key questions:
    1. Are there several projects being considered?
        - If not, the project is acceptable if the NPV is positive.
        - If there are several alternative projects, choose the one with the highest NPV.
    2. Does the community budget limit the size of the project?
        - If so, choose the combination of projects that maximize net benefits, given the community's budget constraints.
        - Example:

	Benefits	*Costs*	*Net Benefits*	*Benefits/Costs*
Project A	10	1	9	10
Project B	22	10	12	2.2
Project C	8	4	4	2
Project D	4	2	2	2
Project E	8	10	-2	0.8

            - In this case, if there is no constraint, choose A, B, C & D
            - If there is a constraint, find the best combination
                - For example, consider a $10 budget
                - Choosing A, C, & D give a net benefit of $15, whereas B alone only gives net benefit of $12.
            - A note on the benefit-cost ratio
                - This is an example of how the benefit/cost ratio doesn't select the project with the highest NPV.
                    - NPV is highest for B, but the benefit/cost ratio is highest for A.
                    - However, when we face a budget constraint, the benefit/cost ratio helps us identify the project that has the most "bang for the buck."
    3. Can the scale of the project be varied?
        - If so, choose the size where marginal benefits = marginal costs.