

# Lecture # 12 -- Production

## I. Production Functions

- Where we are going:
  - Our goal in the next 3 weeks is to derive the supply curve. There are three steps:
    1. Define production
    2. Cost minimization
      - What is the cheapest way to produce a given level of output?
    3. Profit maximization
      - Given cost minimization, how much should we produce?
- We begin by using production functions to describe the production process.
- Firms take inputs and transform them into outputs. A production function describes this process.
- Production function -- indicates the maximum amount of output that can be obtained for any combination of inputs *with a given technology*.
  - Note that changes in technology change the production function.
  - Describes what is *technically feasible* when the firm operates efficiently.

## II. The Short-Run: Production with Fixed and Variable Inputs

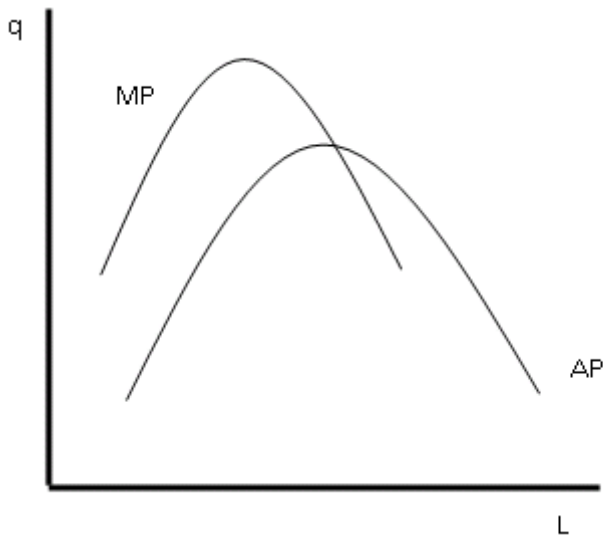
- Inputs can either be fixed or variable
  - Fixed inputs are inputs such as machinery and factories that take time to change
    - In general, we will classify the fixed inputs as capital (K)
  - Variable inputs can be changed easily at any time
    - In general, we will use labor (L) as our variable input
- Short run -- A period of time so brief that at least one factor of production is fixed, and cannot be changed
- Long run -- A period of time long enough so that all inputs can be varied.
- For now, we hold capital constant, and look at what happens as we vary labor. We begin with three definitions:
  - Total product of labor (TP<sub>L</sub>) -- Shows what happens to output when the firm changes one of its inputs, *holding all others constant*.
  - Marginal product of labor (MP<sub>L</sub>) -- Additional output resulting from a one-unit increase in labor, *holding all other inputs constant*. (MP<sub>L</sub> = change in q/change in L)
  - Average product of labor (AP<sub>L</sub>) -- Output per unit of labor. (AP<sub>L</sub> = q/L)

- Note that the marginal product of labor initially rises, and then tends to fall as more and more labor is added.
  - Initially, specialization allows additional workers to be more productive.
  - Eventually, we hit the constraint of fixed inputs. If other inputs are fixed, there is only so much additional labor that can be used.
    - These constraints may be physical, such as a lack of space or equipment, or they may be practical. For example, it may be that the organization gets so large that communication is difficult, so too much time is spent managing the organization, rather than working towards productive output.
    - At the same time, organizations hire the best workers first. Thus, as more and more are hired, the quality of the new workers will begin to suffer.
  - Law of diminishing returns -- As use of an input increases, with other units fixed, the corresponding increases in output will eventually become smaller.
    - Recall the importance of holding technology constant for this analysis.
- We noted that, as labor is added, production typically goes through three stages:
  - Stage I: increasing marginal returns
    - Specialization allows MP to increase
  - Stage II: diminishing returns to labor
    - MP begins to fall
      - However, it is still positive, so total output does increase
  - Stage III: overutilization of fixed inputs
    - MP is negative
      - Ex: firm gets too large for managers and staff to communicate effectively. More time gets spent on organization than production.
      - Because it is negative, we typically won't observe firms here for long. They can get rid of the extra inputs and make themselves better off.

- The chart below shows the numbers we used in class as an example:

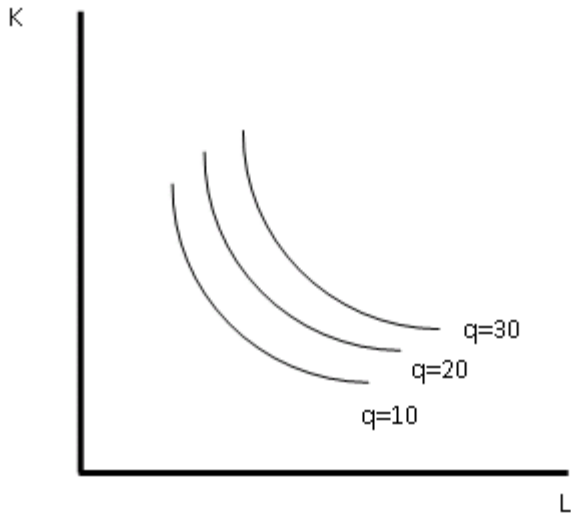
Capital (K)	Labor (L)	Output (Q)	Marginal Product (MP <sub>L</sub> )	Average Product (AP <sub>L</sub> )
10	0	0	--	--
10	1	10	10	10
10	2	30	20	15
10	3	60	30	20
10	4	80	20	20
10	5	95	15	19
10	6	108	13	18
10	7	112	4	16
10	8	112	0	14
10	9	108	-4	12
10	10	100	-8	10

- Finally, we drew graphs of average and marginal product.
  - Remember that when  $AP = MP$ , AP is at its maximum.
    - $MP > AP \Rightarrow AP$  is rising.
    - $MP < AP \Rightarrow AP$  is falling.

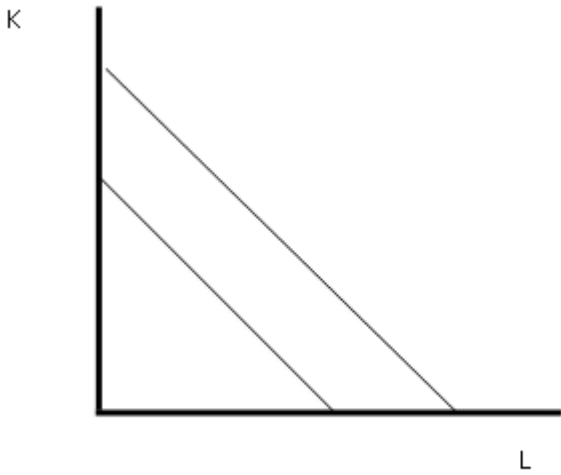


### III. The Long Run: Production with Only Variable Inputs

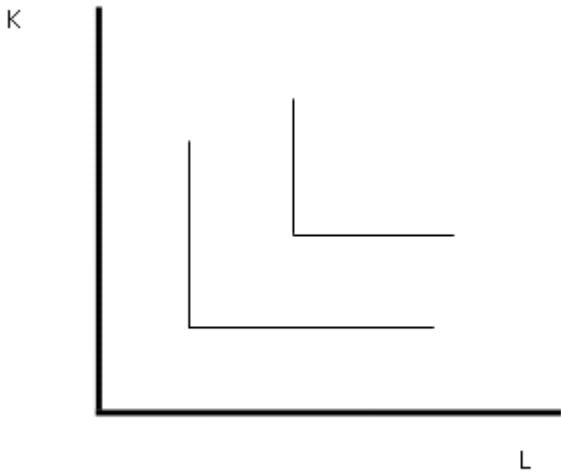
- Isoquant – a curve representing all the combinations of inputs that produce a given quantity of output.
  - Quantity is constant along an isoquant.
  - Quantity is greater on higher isoquants.
  - Isoquants are similar to indifference curves. However, unlike indifference curves, *the number associated with the isoquant matters!*



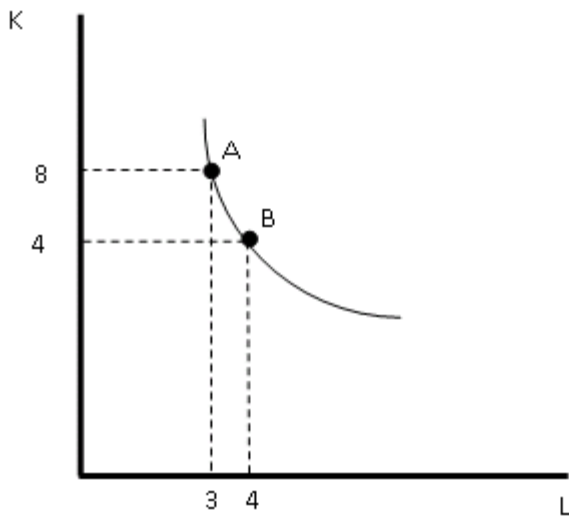
- The shape of the isoquant tells us how easy it is to substitute between inputs.
  - Perfect substitutes: either capital or labor can be used



- Perfect complements: the inputs must be used in fixed proportions (e.g. one driver per bus)



- The Marginal Rate of Technical Substitution (MRTS) of labor for capital is the amount by which capital can be replaced with one extra unit of labor, so that output remains constant.
  - Analogous to the Marginal Rate of Substitution that we used for indifference curves.
  - MRTS is the slope of an isoquant.
  - It falls as we move along an isoquant (because of diminishing marginal products).
  - $MRTS = -\text{change in } K / \text{change in } L = MP_L / MP_K$
  - In this example, we add 1 unit of labor in place of 4 units of capital as we move from A to B. Thus,  $MRTS = 4$ .



#### IV. Returns to Scale

- Returns to scale -- what happens to output when we change *all* inputs by the same proportion.
  - Contrast with marginal product, in which only one input is changed.
- Three possibilities:
  - Increasing returns to scale (IRS): inputs doubled --> output more than doubles.
  - Constant returns to scale (CRS): inputs doubled --> output doubles.
  - Decreasing returns to scale (DRS): inputs doubled --> output less than doubles.
- Note that diminishing marginal product does not necessarily mean decreasing returns to scale.