Appendix to "R&D Subsidies and Climate Policy: Is there a 'Free Lunch'":
Equations and Calibration of the ENTICE-BR Model

This appendix provides further details of the ENTICE-BR model. Section A provides a complete list of equations in the model. Section B discusses calibration of both the base ENTICE model and the ENTICE-BR model, which includes R&D on a backstop technology. Interested readers can learn more about these models in Popp (forthcoming, 2004).

A. Equations of the ENTICE-BR Model:

Exogenous variables and parameters

t = time
L_t = population at time t, also equal to labor inputs
L_0 = initial population level
$g_{L,t}$ = growth rate of population
$g_{L,0}$ = initial value of the growth rate of population
d_L = rate of decline of $g_{L,t}$
D_t = pure time preference discount factor
r_0 = initial value of the pure rate of social time preference
g_r = growth rate of the social time preference
A_t = total factor productivity
A_0 = initial value of total factor productivity
$g_{L,t}$ = growth rate of total factor productivity
$g_{L,0}$ = initial value of the growth rate of total factor productivity
d_L = rate of decline of $g_{A,t}$
γ= elasticity of output with respect to capital
β= elasticity of output with respect to energy/carbon inputs
Φ_t = ratio of carbon emissions per unit of carbon services
$g^z{}_t$ = growth rate of Φ_t per decade
δ = rate of decline of $g^z{}_t$
ζ_1, ζ_2, ζ_3 = parameters of the long-run carbon supply curve
markup= energy services price markup
*CumC** = Total carbon resources available
δ= rate of depreciation of the physical capital stock
δ_H = rate of depreciation of energy knowledge stock
crowdout = percentage of overall R&D crowded out by energy R&D
a, b, ϕ= parameters of the innovation possibilities curve
η = effect of backstop energy knowledge on backstop price
α_H = scaling factor for the stock of energy knowledge
α_Φ = percentage of exogenous carbon intensity reduction
ρ_H = substitution parameter for energy and knowledge
ρ_B = substitution parameter between fossil fuels and backstop energy

LU_t = Land-use carbon emissions
LU_0 = Initial land-use carbon emissions
δ_{LU} = Rate of decline of land-use carbon emissions
ϕ_{11}, ϕ_{12}, ϕ_{21}, ϕ_{22}, ϕ_{23}, ϕ_{32}, ϕ_{33} = Parameters of the carbon transition matrix
O_t = Increase in radioactive forcing over preindustrial levels due to exogenous anthropogenic causes
σ_1, σ_2, σ_3 = Temperature dynamics parameters
θ_1, θ_2 = Parameters of the damage function
$4.1/\lambda$ = Climate sensitivity – equilibrium increase in temperature from a doubling of CO_2 concentrations)

Endogenous Variables

U_t = utility in period t
c_t = per capita consumption
Q_t = output (trillions of 1990 US dollars)
Ω_t = damages from climate change
μ_t = emissions control rate in DICE model
K_t = physical capital stock (trillions of 1990 US dollars)
E_t = energy inputs
$p_{F,t}$ = price of fossil fuels
$p_{B,t}$ = price of backstop energy
F_t = fossil fuel/carbon inputs, also equal to CO_2 emissions
B_t = backstop energy, in carbon ton equivalents (CTE)
q_F = marginal cost of fossil fuel extraction
$CumC_t$ = cumulative carbon extractions by year t
I_t = investment in physical capital
C_t = total consumption
H_{E_t} = stock of energy efficiency knowledge
H_{B_t} = stock of backstop energy knowledge
R_{E_t} = energy R&D
EM_t = Carbon emissions
$M_{A,t}$ = Atmospheric CO_2 concentration
$M_{U,t}$ = Upper oceans/biosphere CO_2 concentration
$M_{L,t}$ = Lower oceans CO_2 concentration
$FORCE_t$ = Radioactive forcing, increase over preindustrial level
T_t = Atmospheric temperature, increase over 1900 level
TL_t = Lower ocean temperature, increase over 1900 level

The ENTICE model maximizes per capita utility, defined in equation (A 1) below, subject to a set of environmental and economic constraints. Economic constraints are represented by equations (A 2) – (A 18). Equations (A 19) – (A 28) are the environmental constraints. In addition to the equations here, imperfect R&D markets are simulated by constraining the returns on energy R&D to be at least four times that of other investments (I_t).

(A 1) $$\max V = \sum_{t=0}^{T} U[c_t, L_t] D_t$$

Economic Constraints

(A 2) $$U_t = L_t \log(C_t / L_t)$$

(A 3) $$D_t = \prod_{v=0}^{t} \left[1 + r_0 e^{-g_r t}\right]^{10}$$

(A 4) $$Q_t = \Omega_t (A_t K_t^{\gamma} L_t^{1-\gamma-\beta} E_t^{\beta}) - p_{F,t} F_t - p_{B,t} B_t$$

(A 5) $$K_t = \{I_t - 4 * crowdout * (R_{E,t} + R_{B,t})\} + (1-\delta) K_{t-1}.$$

(A 6) $$L_t = L_0 \exp(g_{L,t})$$

(A 7) $$g_{L,t} = (g_{L,0}/d_L) * (1-\exp(-d_L * t))$$

(A 8) $$A_t = A_0 \exp(g_{A,t})$$

(A 9) $$g_{A,t} = (g_{A,0}/d_A) * (1-\exp(-d_A * t))$$

(A 10) $$E_t = \left[\alpha_H H_{E,t}^{\rho_H} + \left(\left(\frac{F_t}{\alpha_\Phi \Phi_t} \right)^{\rho_B} + B_t^{\rho_B} \right)^{\rho_H / \rho_B} \right]^{1/\rho_H}, \qquad \rho \le 1$$

$$\Phi_t = \exp\left[\left(\frac{g_t^z}{\delta^z} \right) \left(1 - \exp\left(-\delta^z t\right)\right) \right]$$

(A 11) $$P_F = q_F + markup$$

(A 12) $$q_F = \zeta_1 + \zeta_2 [CumC_t / CumC^*]^{\zeta_3}$$

(A 13) $$CumC_t = CumC_{t-1} + 10 * F_t$$

(A 14) $$F_t < 0.1 * (CarbMax - CumC_t)/10$$

(A 15) $$p_{B,t} = \frac{P_{B,0}}{H_{B,t}^{\eta}}$$

(A 16) $$H_{i,t} = h(R_{i,t}) + (1-\delta_H) H_{i,t-1}, \qquad i = E,B$$

(A 17) $$h(R_{i,t}) = a R^b_{i,t} H^{\phi}_{i,t}, \qquad i = E,B$$

(A 18) $$Q_t = C_t + I_t + R_{E_t} + R_{B_t}$$

Environmental Constraints

(A 19) $LU_t = LU_0(1-\delta_{LU})^t$

(A 20) $EM_t = F_t + LU_t$

(A 21) $M_{A,t} = 10*EM_t + \phi_{33}M_{L,t\text{-}1} + \phi_{23}M_{U,t\text{-}1}$

(A 22) $M_{L,t} = \phi_{11}M_{A,t\text{-}1} + \phi_{21}M_{U,t\text{-}1}$

(A 23) $M_{U,t} = \phi_{12}M_{A,t\text{-}1} + \phi_{22}M_{U,t\text{-}1} + \phi_{32}M_{L,t\text{-}1}$

(A 24) $FORCE_t = 4.1*\{\log(M_{A,t}/596.4)/\log(2)\} + O_t$

(A 25) $O_t = -0.1965 + 0.13465t, \quad t < 11$

$O_t = 1.15, \quad t \geq 11$

(A 26) $T_t = T_{t\text{-}1} + \sigma_1\{FORCE_t - \lambda T_{t\text{-}1} - \sigma_2(T_{t\text{-}1} - TL_{t\text{-}1})\}$

(A 27) $TL_t = TL_{t\text{-}1} + \sigma_3(T_{t\text{-}1} - TL_{t\text{-}1})$

(A 28) $\Omega_t = 1/(1 + \mathrm{a}_1 + T_t + \mathrm{a}_2*T_t^2)$

B. Calibration of the ENTICE-BR Model:

This appendix describes the steps taken to calibrate the ENTICE-BR model. I begin by summarizing calibration of the ENTICE model without a backstop technology, followed by a discussion of changes necessary to incorporate the backstop technology.

As a global macroeconomic model, ENTICE uses Nordhaus' DICE model (1994, Nordhaus and Boyer 2000) as its basic building block. Since the current version of Nordhaus' DICE model does not include carbon emissions as an input, but rather simply models emissions as a byproduct of output requiring control, the first step to constructing the model is to add a fossil fuel sector that mimics the behavior of the original DICE model. I do this using the same modeling structure as Nordhaus' RICE model, except that I apply the equations at a global, rather than regional, level. The complete equations of the model are presented above. I calibrate this basic model, with no energy R&D, so that the results are comparable to Nordhaus' DICE model. To begin, I take the initial value of *F* from the latest version of the DICE model. I then solve for initial values of A and K that reproduce the initial output found in the DICE model. Next, I calculate the elasticity of output to with respect to energy, β, as the percentage of output spent on fossil fuels in the initial period, using the 1995 price of carbon based on equations (A11) and (A12). Finally, the growth rate of Φ, g^{z} (-15.49), and the rate of decline of this growth rate, $\gamma^{\bar{z}}$ (23.96), are chosen to produce an emissions path as close as possible to the DICE model. These values represent the rate of exogenous decline in carbon intensity without any energy R&D in the model. Figures A1 and A2 compare the emissions and output that result from this calibration.

Having added carbon fuels as an input to production in the DICE model, the next step is to add induced technological change to the ENTICE model. The modeling for this stage is described the main text of the paper. Calibration requires choosing values for the following parameters:

- the initial value of energy research, R_{E0}.
- ρ_H, the substitution parameter in equation (A10),
- parameters in the invention possibilities frontier (A17): a, b, and ϕ, and
- the initial level of energy human capital, H_{E0},[1]
- α_H, the scaling factor for the effect of this human capital, and
- α_Φ, the percentage of exogenous technological change remaining.

To calibrate the energy R&D sector, three goals must be met. First, R&D levels should be consistent with historical levels. A starting value of $10 billion is chosen for the base year of 1995. To get this value, I begin with an estimated level of total global spending on R&D of $500 billion. This figure is based largely on data from OECD countries. Energy R&D data is not

[1] Note that, since human capital enters the invention possibilities frontier multiplicatively, the initial value cannot be zero.

available on a global basis. However, it is available for the United States. In the U.S., two percent of R&D spending in 1995 went to energy-related R&D. The $10 billion figure used in this paper is simply two percent of the global level of R&D. This figure is also close to the initial value of R&D used by Nordhaus (2002).

Second, the behavior of energy R&D should be consistent with empirical studies both *across time* and *across policy dimensions*. Based on Popp (2002), I use an elasticity of energy R&D with respect to energy prices of 0.35 for the base model. As the price of carbon rises over time, the time path of energy R&D should follow the path predicted by this value as closely as possible.[2] In addition, elasticities of energy R&D calculated on differences in the carbon price with and without a carbon tax in the optimal policy simulation should also equal 0.35. Since the goal of this paper is to explore the consequences of omitting endogenous technological change from earlier climate change models, when these two goals are incompatible, the second takes precedence. Furthermore, since Popp (2002) also notes that energy R&D experiences diminishing returns over time, the calibrated elasticity should fall over time. Figure A3 shows the calibrated levels of energy R&D and what would be predicted by a constant elasticity over time of 0.35.

Finally, Popp (2001) estimates a 4:1 ratio on the returns to energy R&D. Thus, each dollar of energy R&D should lead to a four dollar reduction in energy savings. The model is calibrated so that a weighted average of energy savings each period (weighted by the discount factors used in the model) produce a 4:1 ratio of energy savings to energy R&D.

Using these goals as guidelines for choosing the parameters, I first choose the value of HE_0 to approximate baseline emissions in early years of the simulation. This value is 0.0001.

[2] Note that, to account for growth in the level of economic activity, all elasticities are calculated based on a ratio of energy R&D to global output.

Next, I choose ρ_H to approximate the elasticity of energy R&D between the no-policy and optimal policy simulations. This value is 0.38. Third, I set the value of the scaling factor α_H to 0.336 to yield the appropriate rate of return on energy R&D. To calibrate the inventions possibility frontier, the value a is chosen so that the change in energy R&D between 1995 and 2005 in the optimal policy simulation is consistent with the elasticity of 0.35. Values of b and ϕ are chosen so that future elasticities fit the desired time path – falling slowly in the near future due to diminishing returns to R&D. Once the desired time path of R&D is calibrated, the scaling factor α_Φ can be adjusted to change the level of baseline emissions as appropriate. A value of 0.8 is used in the base model, meaning that 80 percent of exogenous technological change remains in the ENTICE model. As a result, purposeful R&D efforts to improve energy efficiency are only a small portion of the changes that take place over time to reduce energy intensity. Table 1 in Popp (2004a) presents a complete list of the parameter values chosen for both the base model and various sensitivity analysis scenarios of the base ENTICE model.

When adding a backstop, the first critical piece of information is the initial conditions. Based on Nakicenovic *et al*. (1998), the backstop technology is assumed to contribute four percent of total energy in 1995. This yields an initial backstop level of 0.25 carbon ton equivalents (CTE). The parameter β from the production function, which equals the share of energy expenses taken from output, is adjusted accordingly, as the share of production costs going to energy is now greater. To be consistent with R&D data (Anderson 1997), the initial level of backstop energy R&D is ten percent of energy efficiency R&D, or $1 billion. The initial stock of backstop knowledge, $H_{B,0}$ is normalized to 1.

As with energy efficiency R&D, the value of ρ_B has a significant impact on the elasticity of backstop energy R&D. However, its value is not set independently. Based on the first-order conditions for energy demand, ρ_B is determined by initial energy consumption and the relative prices of fossil fuels and the backstop technology. Unfortunately, a wide range of possibilities for the starting price exists. Popp (forthcoming) presents sensitivity analysis for three initial price levels. The first is an initial price of \$400 per carbon ton equivalent (CTE) of backstop energy. This is based on a study by Burtraw *et al.* (1995), who report the cost of wind energy to be 44% higher than that of energy from fossil fuels. Gerlagh and Lise (2003) report prices for alternative energy sources ranging from 2 to 5 times that of fossil fuels. Using the upper range of this as an alternative, I consider an initial price of \$1200 as a second option. Finally, as noted in Popp (2004b), the resulting elasticity of substitution (ρ_B) using these prices yields very high elasticities of R&D in each case. Thus, I also consider a starting price of \$2000 CTE. This provides more reasonable elasticities of backstop energy R&D, as the resulting elasticity of substitution is similar to that for energy efficiency R&D.[3] In this paper, only the mid-range value of \$1200 is used.

Next, a value for η, which relates human capital to backstop price decreases, is chosen. Again, no good empirical estimates exist. Results for two values, 0.5 and 1.0, are presented in Popp (forthcoming). These yield progress ratios of 24 and 50 percent respectively. A 50 percent progress means that a doubling of the knowledge stock reduces the backstop price by 50 percent. More importantly, under realistic base case R&D scenarios, the resulting time paths for the share of energy consumption from backstop energy R&D are comparable to other studies. Such rapid

[3] To compare these prices to existing estimates of renewable energy costs, it is useful to convert the prices to cents per kWh. Using data on total primary energy supply (IEA 1997), I calculate the energy services provided per ton of carbon emission. Based on the initial carbon price of \$276.29, this yields a cost of energy of 1.8 ¢/kWh (in 1990 dollars). In comparison, the backstop costs used in the model imply costs of 2.4¢/kWh, 7.1¢/kWh, and 11.9¢/kWh, respectively. Such estimates are in the range of estimated renewable costs provided in the literature (see, for example, Table 7.25 in Goldemberg *et al*. (2000). Finally, although the high-price scenario is at the upper level of renewable price estimates, keep in mind that, for the elasticity of substitution, what matters is the price of the last backstop energy unit consumed. One would expect this to be higher than prices for technologies in ideal environments.

progress is comparable to changes in patenting and prices during the past 20 years. The 24 percent progress ratio yields slightly lower shares of backstop energy than comparable scenarios. However, as shown in the results section, the marginal returns to R&D are more realistic. Thus, a 24 percent progress ratio is used in this paper.

Finally, the parameters of the inventions possibilities frontier are chosen as before. At the same time, the parameters *a* and *b* for energy efficiency R&D are changed slightly so that base case R&D is comparable in simulations with and without a backstop technology. Table A1 provides a list of the parameters needed for the various trials of the ENTICE-BR model used in this paper. Figures A4 and A5 show how backstop energy R&D and energy efficiency R&D vary in under an optimal climate policy (without subsidies) vary under these different assumptions.

Appendix References

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Figure A1 – Industrial Emissions in the ENTICE & RICE Models

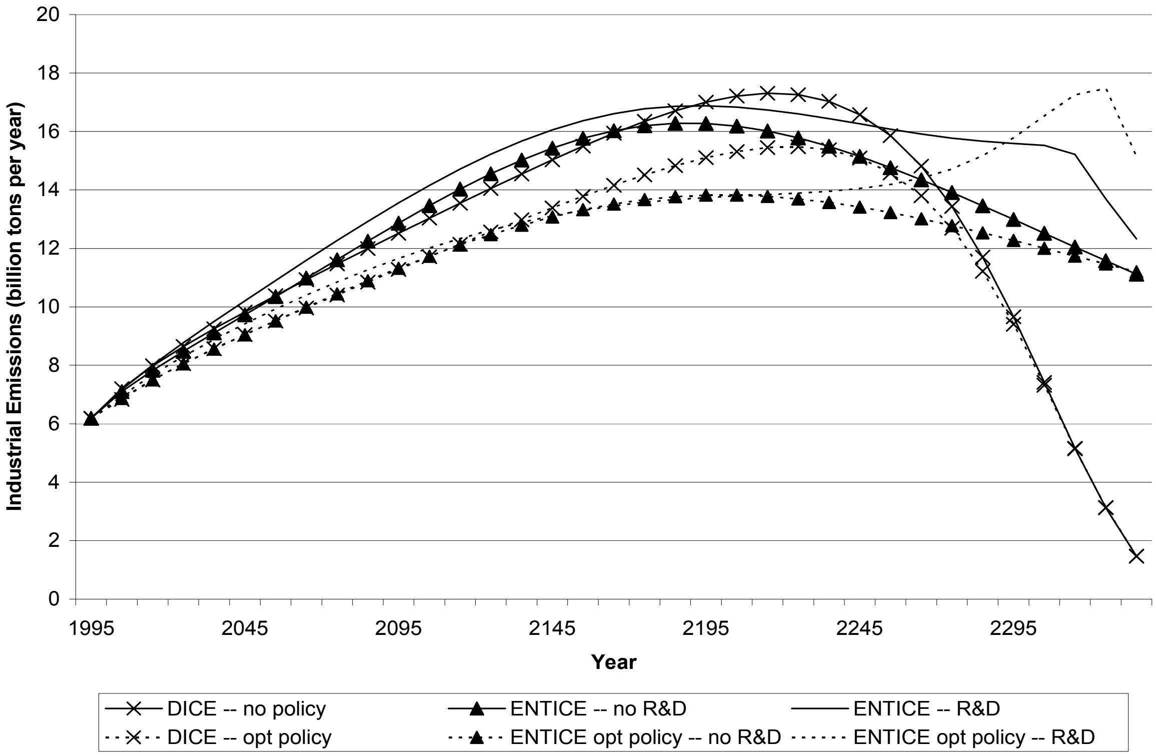


Figure A2 – Output in the ENTICE & RICE Models

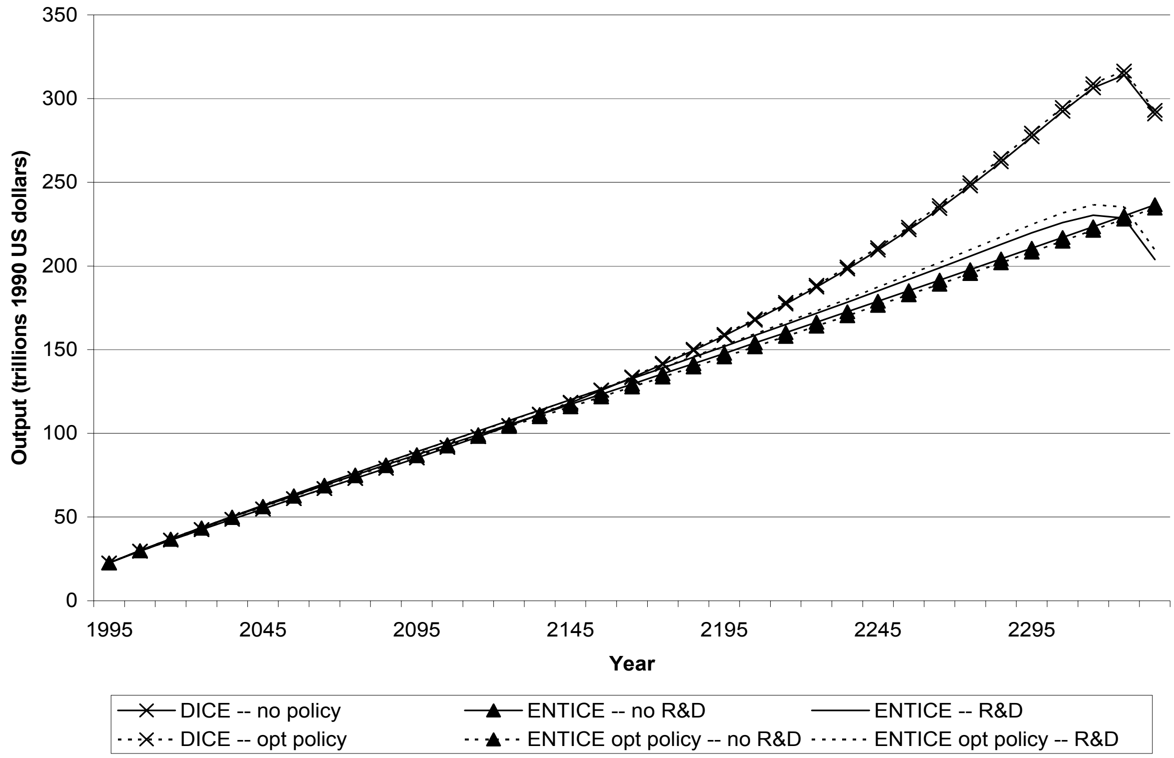


Figure A3 – Predicted and Actual Energy R&D

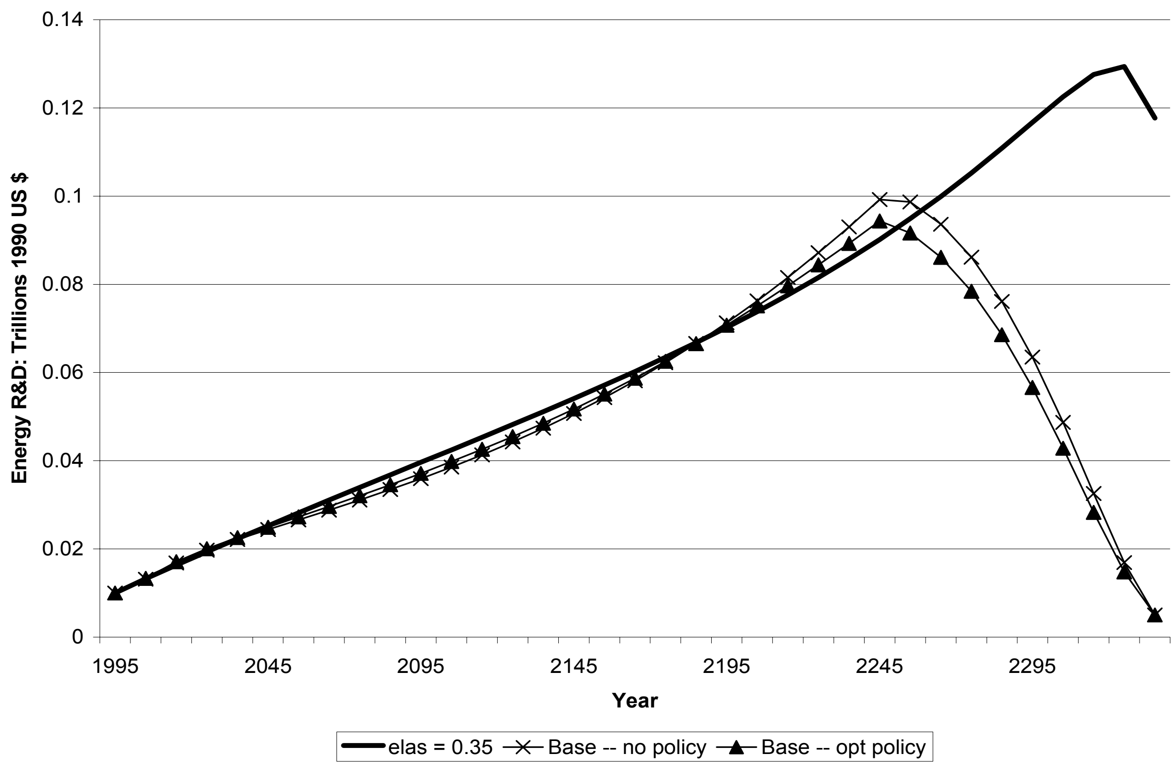
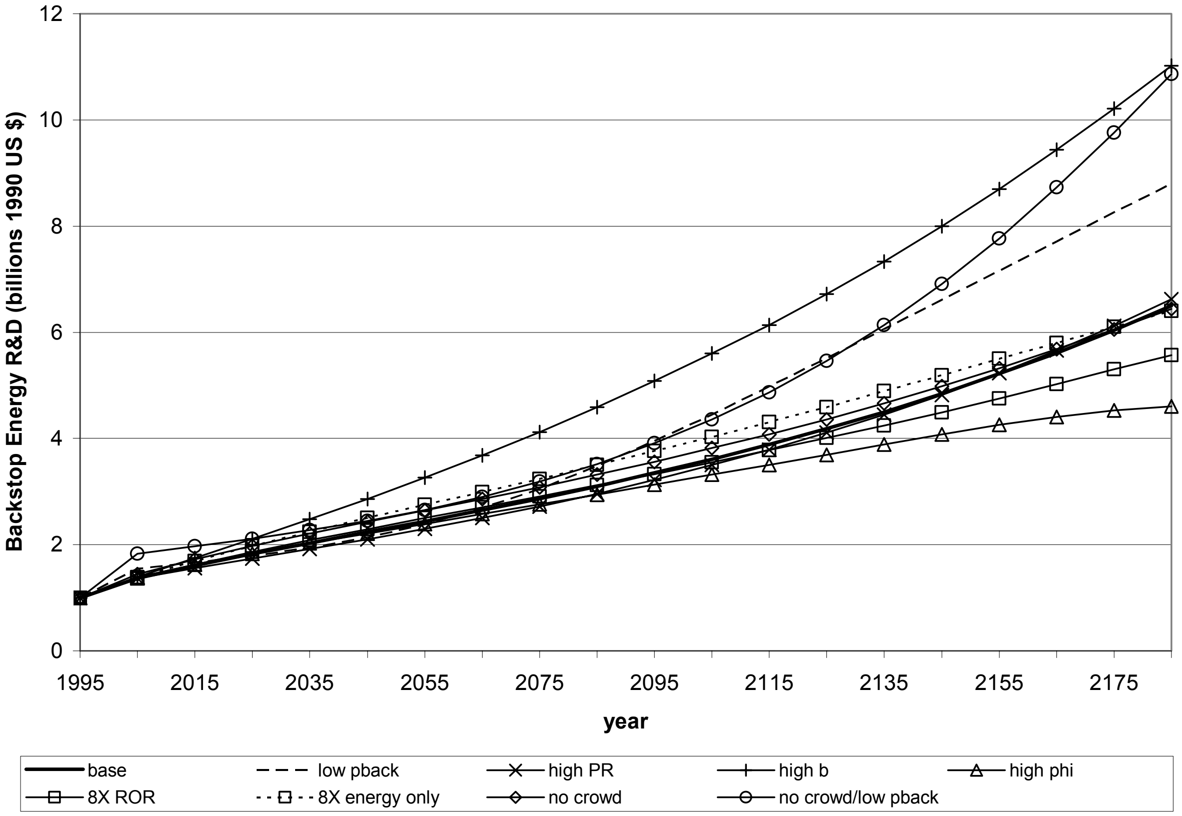
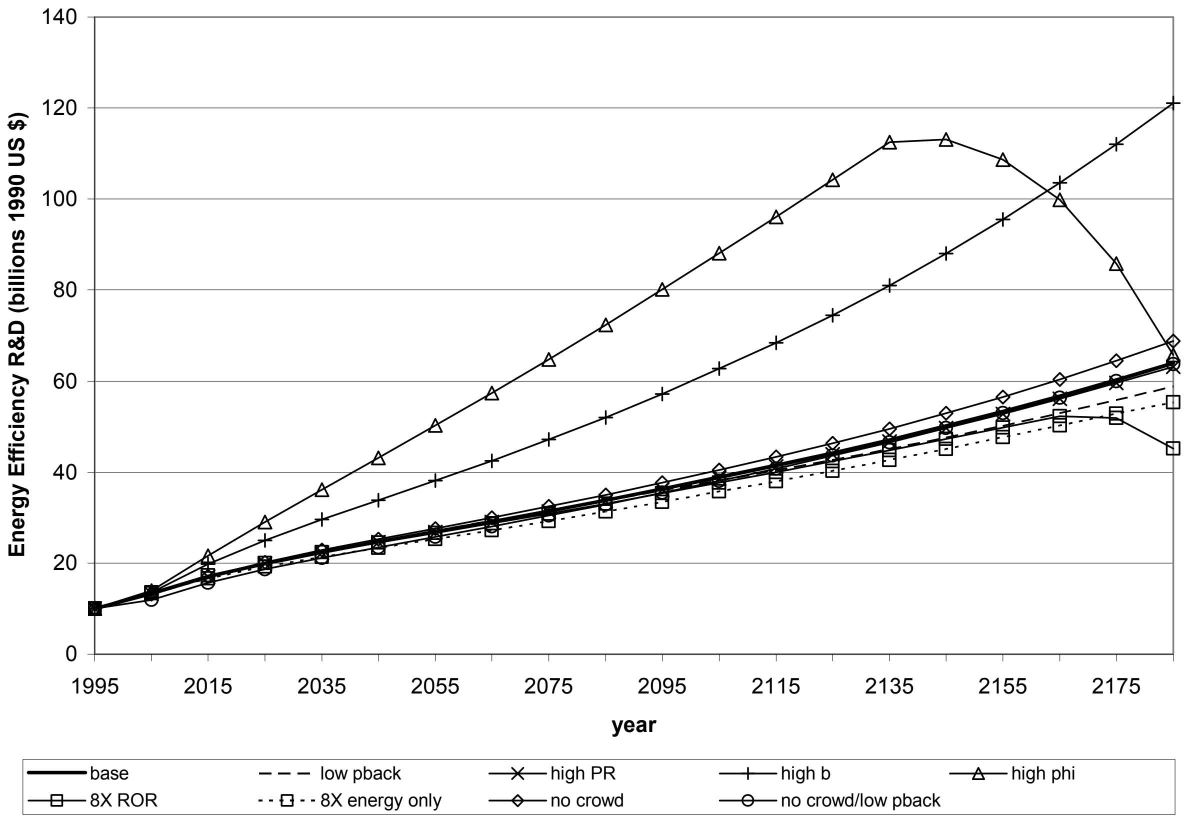


Figure A4 – Backstop Energy R&D Sensitivity



The figure shows how backstop energy R&D in the optimal policy simulation (without subsidies) changes in the various sensitivity trials.

Figure A5 – Energy Efficiency R&D Sensitivity



The figure shows how energy efficiency R&D in the optimal policy simulation (without subsidies) changes in the various sensitivity trials.

Table A1 – Parameters for the Base Case of ENTICE-BR

	backstop parameters						*energy efficiency IPF*			*R&D markets*	
Trial	initial price	effect of technology on backstop price: η	IPF: *a*	IPF: *b*	IPF: ϕ	sub between backstop/fossil fuels	IPF: *a*	IPF: *b*	IPF: ϕ	ROR constraint	Crowding Out
base	1200	0.4	0.0122	0.1	0.54	0.542	0.0264	0.2	0.54	4X	Yes
low pback	400	0.4	0.0155	0.0525	0.55	0.885	0.0286	0.2	0.55	4X	Yes
high PR	1200	1	0.00505	0.073	0.54	0.542	0.0264	0.2	0.54	4X	Yes
high b	1200	0.4	0.011	0.2	0.54	0.542	0.0298	0.4	0.54	4X	Yes
high phi	1200	0.4	0.0122	0.1	0.65	0.542	0.06	0.2	0.65	4X	Yes
8X ROR	1200	0.4	0.02	0.18	0.55	0.542	0.062	0.2	0.55	8X	Yes
8X energy only	1200	0.4	0.0137	0.25	0.54	0.542	0.0562	0.2	0.54	8X energy	Yes
no crowd	1200	0.4	0.004	0.2	0.54	0.542	0.026	0.2	0.54	4X	No
no cr./low pb.	400	0.4	0.0065	0.1	0.55	0.885	0.026	0.2	0.55	4X	No