PAI 897 **Professor David Popp**
Solutions to Problem Set #4 **Fall 2023**

1. Begin with what we know. First, when there is 0 labor, total product must be equal to 0, which we see from the average product (AP = TP/L). When there is 1 unit of labor, total product is 180. Thus, average product equals 180, and marginal product = 180.

Now, proceed to 2 units of labor. The marginal product is 140. Thus, output increases by 140 units when a second worker is added. Total product thus equals 180 + 140, or 320. Average product equals 320/2, or 160.

With three workers, we are given total product, which equals 420. Average product equals 420/3, or 140. Marginal product is the change in output as labor increased from 2 to 3, which equals 100.

Finally, for 4 workers we are given average product. We can find total product by multiplying average product by the amount of labor. Thus, total product equals 120 x 4, or 480. To get marginal product, we need the change in total product when the fourth worker was added. This equals 60.

Labor	Total Product	Average Product	Marginal Product
0	0	**0**	**--**
1	**180**	180	180
2	320	160	**140**
3	420	140	100
4	**480**	**120**	60

(note: bold numbers are those given in the problem)

2. For this question, you need to compare the *marginal product* of police officers in each of the two districts. Although the average products are equal in each neighborhood, their marginal products are not. Currently, there are 200 officers in Center City, and their marginal product equals 35 arrests. There are 300 officers in West Philadelphia, and their marginal product equals 40 arrests. If we took 100 officers out of Center City, arrests in that neighborhood would fall by 35. If we redeploy those same officers in West Philadelphia, there will be 40 more arrests in West Philadelphia. The net increase in arrests is 5. Once this is done, no further improvements are possible, since the marginal product in West Philadelphia is still 40, but the marginal product in Center City is now 45.

Another way to think about this question is to ask where each block of 100 police would be most effective. The first 100 police are most effective in Center City: the marginal product is 45, rather than 40 if placed in West Philadelphia. After that, the marginal product will always be higher placing police in West Philadelphia, rather than in Center City.

While it isn't necessary to answer the question (and, indeed, you should answer the question by looking at marginal product, rather than calculating the results for each possible combination), note that the total number of arrests from having 300 police in West Philadelphia and 200 police in Center City is 200 (=120+80). The total number of arrests from having 400 in West Philadelphia and 100 in Center City is 205 (=160+45). Thus, arrests go up by 5, as predicted above.

3. The director's analysis does not make sense. He is confusing average and marginal costs. The $10 per hour figure is an average cost. However, what matters here is marginal cost. That is, how much more will it cost Hot Meals to hire one more worker? That cost is $20 per hour. It is that marginal cost that should be compared to the marginal benefits of hiring this worker to decide if it is worth adding more staff members.

Note that the key concept here is identifying the appropriate marginal cost. Some students answered by mentioning opportunity cost. While opportunity costs are relevant, identifying what the true opportunity cost is (e.g. giving up $20 per hour for each new employee) is still important. Similarly, noting that the director is ignoring marginal revenue is also not relevant. That assumes the goal of Hot Meals is to maximize profits. Is that necessarily the case for a non-profit organization?

4. a) Lift Up is minimizing costs. To see this, we need to compare the marginal product per dollar for each type of employee. Professional staff work on 15 cases per hour, but earn $30 per hour. Thus, they work on 0.5 cases per dollar earned (= 15/30). While high school students only prepare 5 cases per hour, they earn just $10 per hour. Thus, they also prepare 0.5 cases per dollar earned (= 5/10). Thus, spending one dollar on professional staff yields the same number of cases as one dollar spent on high school workers. There is no way to reallocate resources and increase output. Given this, Lift Up is minimizing costs.

Note that having equal marginal products per dollar doesn't mean that the number of workers must be equal. We always want the marginal product per dollar to be equal. That doesn't mean that we always want to have equal numbers of each type of worker. If you change the ratio of the workers, the marginal products will change. They are only valid for this combination. The quantities are simply extra information.

b) A $5 subsidy for each student lowers the cost of high school students to $5/hour. Now, a dollar spent on high school workers leads to one completed case (= 5/5). Thus, a dollar spent on high school staff is more valuable than a dollar spent on professional staff. The organization should replace some professional staff with high school workers.

Note that, because there is no additional demand for services, it is not sufficient to say that the organization should just hire more high school staff. If they do so, they would have enough workers to increase output, but no demand for the services. Thus, some people would go unused, and thus would be a waste of money.

Similarly, just because the marginal productivity of a high school worker is now greater than that of professional staff does not mean that the organization should *only* use high school workers. Because of diminishing returns, as more high school workers are hired, their marginal product will decrease. For example, these workers need supervision, and there would be no one to supervise them! Similarly, as you get rid of professional staff, the marginal product of the remaining professional staff will increase. Costs are minimized when the marginal product per dollar spent on each worker are once again equal.

5. Our choice for each community should depend on which type of service provision reduces test scores the most per dollar spent. For this, we need to consider both the costs and the potential test score increase. We compare MP/P for this evaluation.

The marginal product of a year of on-line instruction is 5 points. Each year of instruction costs \$500. Thus, $MP_{on\text{-}line}/P_{on\text{-}line}$ = 5/00 = 0.01. We can then do the same calculation for in-person services in each community. If the marginal product per dollar for in-person services is greater than 0.01, we should use in-person services in that community. Here are the calculations:

Apple Hill:	10/2000 = 0.005
Baskerville:	60/5000 = 0.012
Canterbury:	45/3000 = 0.015
Denali:	50/6000 = 0.0083

Based on these figures, we should use in-person services in Baskerville and Canterbury, and on-line services in the remaining communities.

Alternatively, some students calculated the cost per dollar of test score increase (e.g. P/MP). That is fine as long as you interpret the results correctly. In that case, it costs \$100 per one point increase on-line (500/5). In contrast, the costs per point increase in person are:

Apple Hill:	2000/10 = \$200
Baskerville:	5000/60 = \$83.33
Canterbury:	3000/45 = \$66.67
Denali:	6000/50 = \$120

If you used this method, you want to choose the lowest cost alternative. Thus, you would provide in person services in the communities that cost less than \$100 per dollar of point increase.

In either case, note how both the marginal product and costs are important for this answer. Costs are high in Baskerville and Denali. Nonetheless, the largest increase in test scores also occurs in Baskerville, making it worth providing in-person services despite the higher costs.