PAI 897 **Professor David Popp**
Solutions to Problem Set #3 **Fall 2023**

1. a) By looking at the graph, we can tell utility is not maximized because the indifference curve is not tangent to the budget constraint. Algebraically, we can tell utility is not maximized because marginal utility per dollar spent on each good (MU/P) is not equal.

b) Currently, the marginal utility per dollar spent on y is higher than for x. Intuitively, that means that we are getting more "bang for our buck" from consuming y, rather than x. As a result, we should consume less x, which will result in more y being consumed (since income is being held constant). For example, if we spend $1 less on x, utility will fall by $0.29. However, if we then spend $1 more on y, utility will increase by $0.87. Thus, the combined effect of spending $1 less on x and $1 more on y is that utility will increase by $0.58.

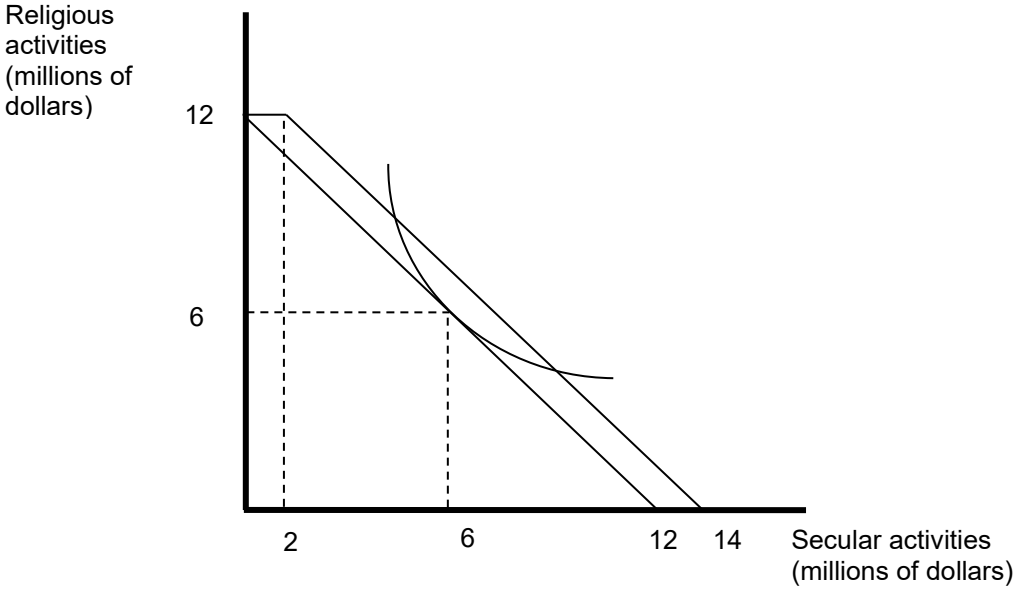
d) If you decreased consumption of x, the marginal utility per dollar spent on x should have increased. This occurs because of diminishing marginal utility. Since you now have less x, the remaining units are more valuable to you.

e) Similarly, as you decreased consumption of x, consumption of y increased. Thus, marginal utility per dollar of y is now lower. Since you have more, additional units of y are not as valuable to you as they were before.

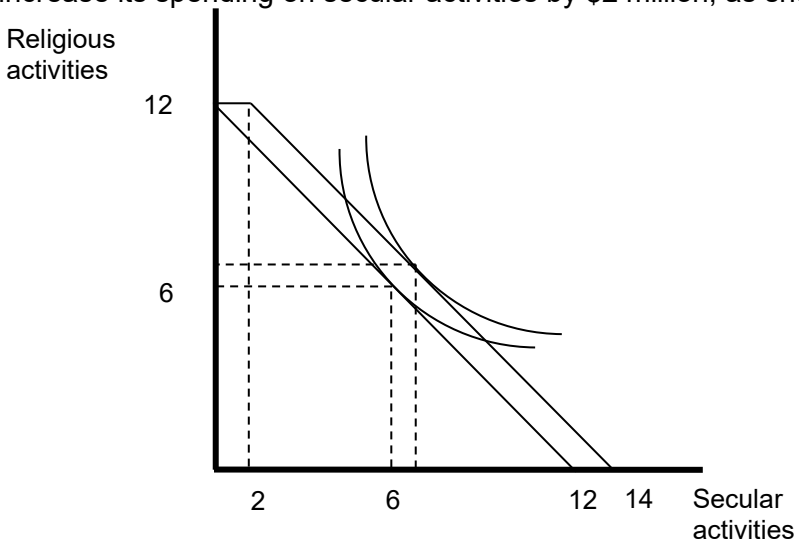
f) Utility is maximized when the marginal utilities per dollar are equal. At this point, it is no longer possible to swap consumption of one good for consumption of the other and still have utility increase. Graphically, the indifference curve is tangent to the budget constraint. Given the initial income and prices, utility is maximized when 10 of each good is consumed.

2. a) Since the federal grant can only be spent on secular activities, Notre Dame can still spent, at most, only $12 million on religious activities. However, should it choose to do so, it will also have $2 million available for secular activities. Alternatively, if Notre Dame chose to spend all available money on secular activities, it would now have $14 million available (its initial $12 million budget plus the $2 million grant). Thus, Notre Dame can spend up to $14 million on secular activities.

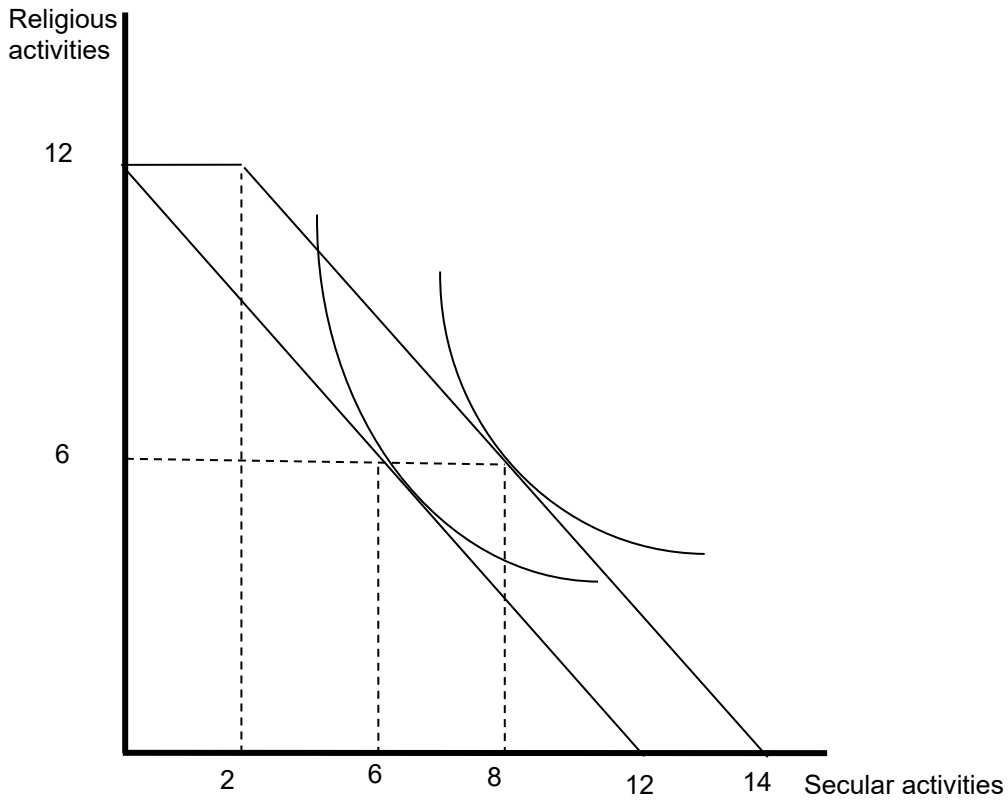
The resulting budget constraint is shown below. Note that the budget constraint shifts out, but is cut off above $12 million of religious activity. Thus, the constraint is similar to the food stamp constraint that we discussed in class.



b) Given the current preferences, there is no reason to believe that the University's welfare (or its behavior) would change if the grant came without the secular use restriction. Despite the restriction, the University of Notre Dame will be able to increase both its secular and religious activities, since the University can simply spend more of its other income on religious activities, while using the entire $2 million from the grant for secular activities. That is, Notre Dame is not forced to increase its spending on secular activities by $2 million, as shown below:



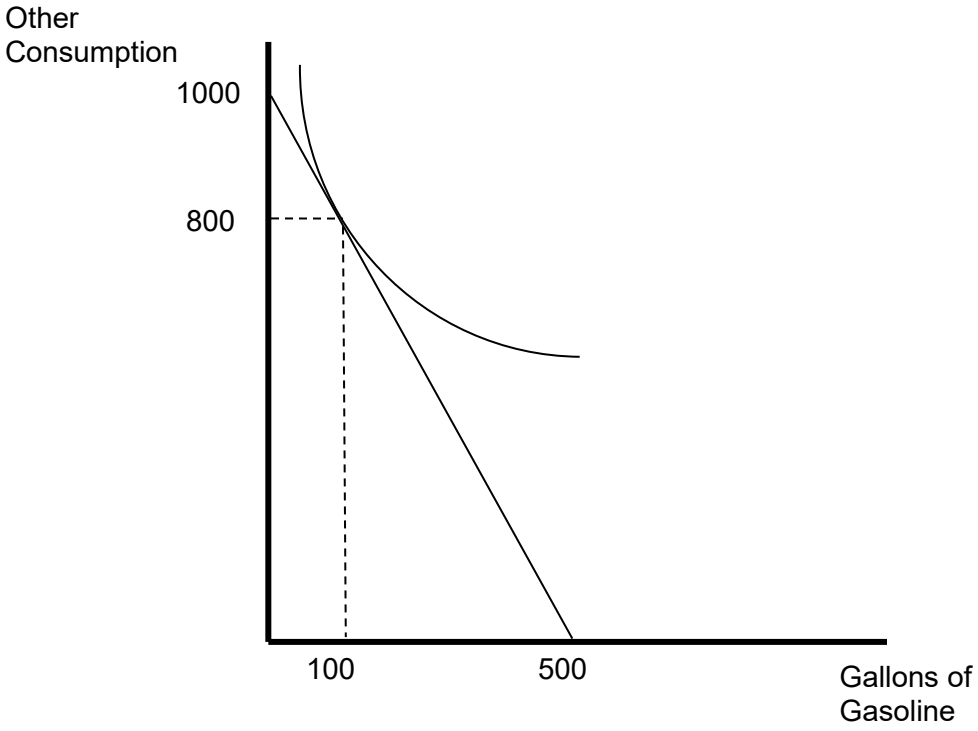
A common answer was to simply draw indifference curves in the middle of the diagram that only showed secular activities increasing:



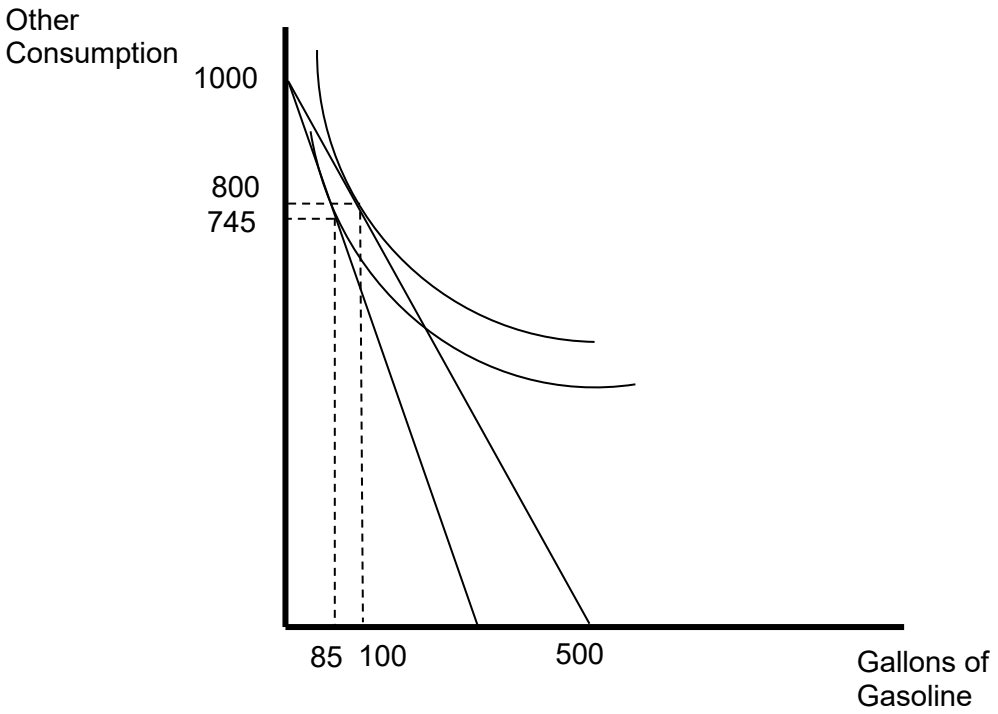
Since the graph is feasible (as long as you've drawn indifference curves that will not cross if extended further), I gave partial credit for this answer. However, I did not give full credit, as such a graph depends on an assumption that is unlikely to hold.

Recall that at the optimal point, $MU_S/P_S = MU_R/P_R$. This must hold at both tangencies above. Since the grant does not change the prices, the only way these can still be equal at the new is if the marginal utility of secular activities, MU_S, does not change as Notre Dame country spends more on secular activities – that is, if there are no diminishing returns to adding more secular activities. Such an outcome is unlikely. For example, suppose they use the extra funds to support new extracurricular activities for students. As more activities become available, students with limited time available will need to choose which activities to do – they won't have time for everything.

3. a) To draw the budget constraint, note that consumers can buy up to 500 gallons of gasoline (= \$1,000/\$2), or \$1,000 worth of other goods. Consumers actually choose to consume 100 gallons of gasoline. Note that these endpoints are what is important for the budget constraint – we want to show *what is possible*, not just what the consumers actually do. Since this cost them \$200, they have \$800 left for other goods. This is shown by drawing an indifference curve tangent to the budget constraint at 100 gallons. This is the highest possible indifference curve given the budget constraint. Intuitively, what is true here is that the marginal utility per dollar of gasoline equals the marginal utility per dollar spent on other consumption. It is not possible for this consumer to move consumption from one good to another and increase utility.

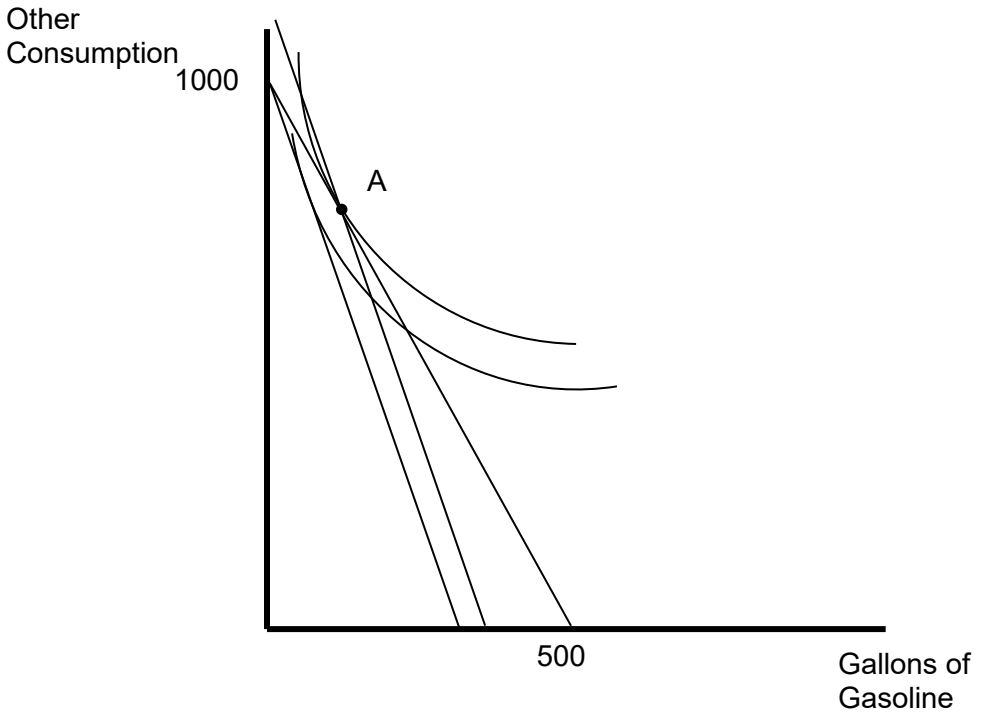


b) The price increase rotates the budget constraint in. Now, the most gasoline the consumer can afford is 333 gallons. For a family that now purchases 85 gallons of gasoline, they can afford to purchase $745 worth of other goods (= $1000 – 3(85)). The new indifference curve, tangent to the new budget constraint, is lower than the original indifference curve. Utility has fallen.

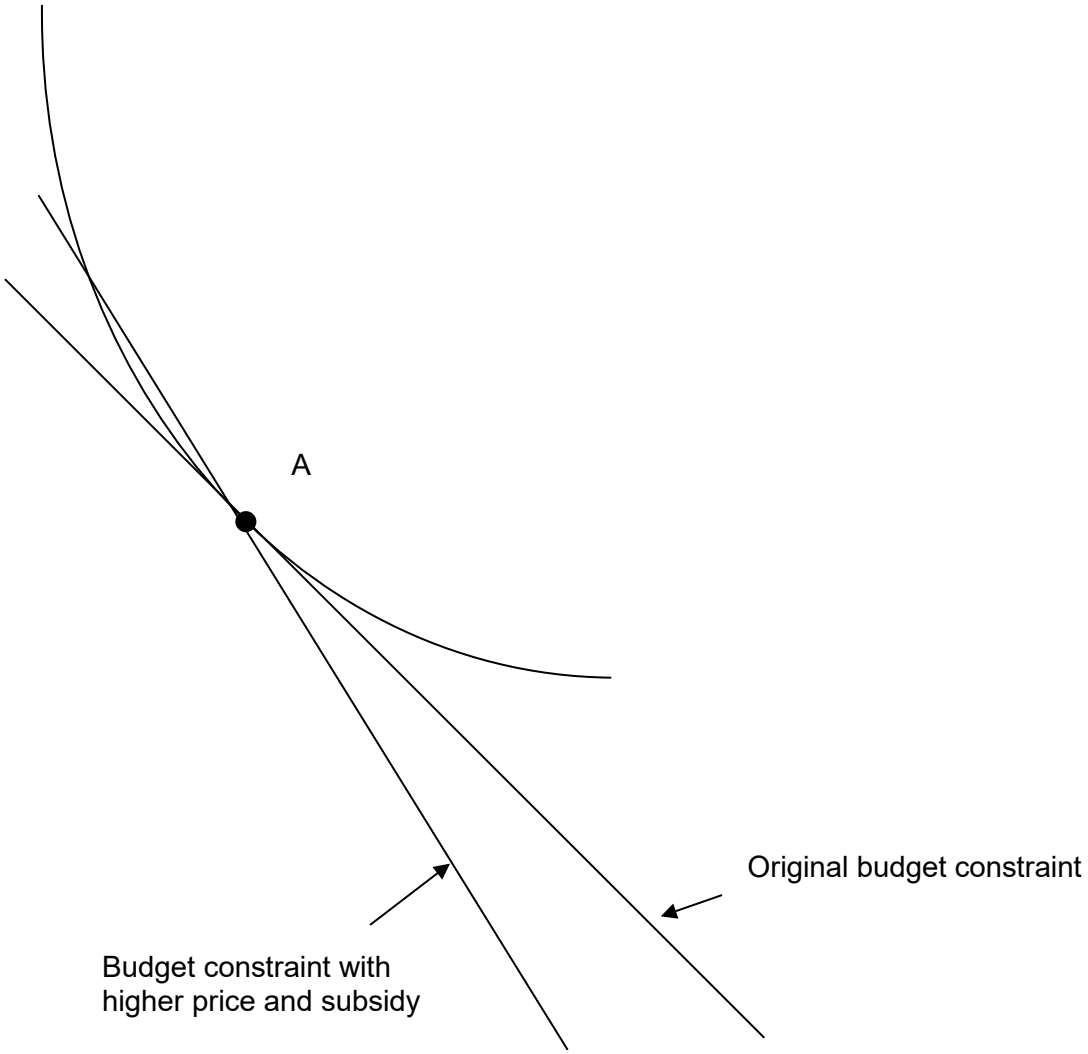


c) To be able to afford 100 gallons of gasoline and $800 of other stuff, the family will need an income of $1100 (= $3X100 + $800). The new budget constraint is parallel to the budget constraint with higher prices, *but goes through the original indifference curve at point A*, which is the original bundle.

Note that this new budget constraint goes through the indifference curve. It is not tangent to it, as is the case when deriving substitution and income effects. The problem here is that the government is over-compensating this family, because it is ignoring the substitution effect. Utility will be higher than it was in January.

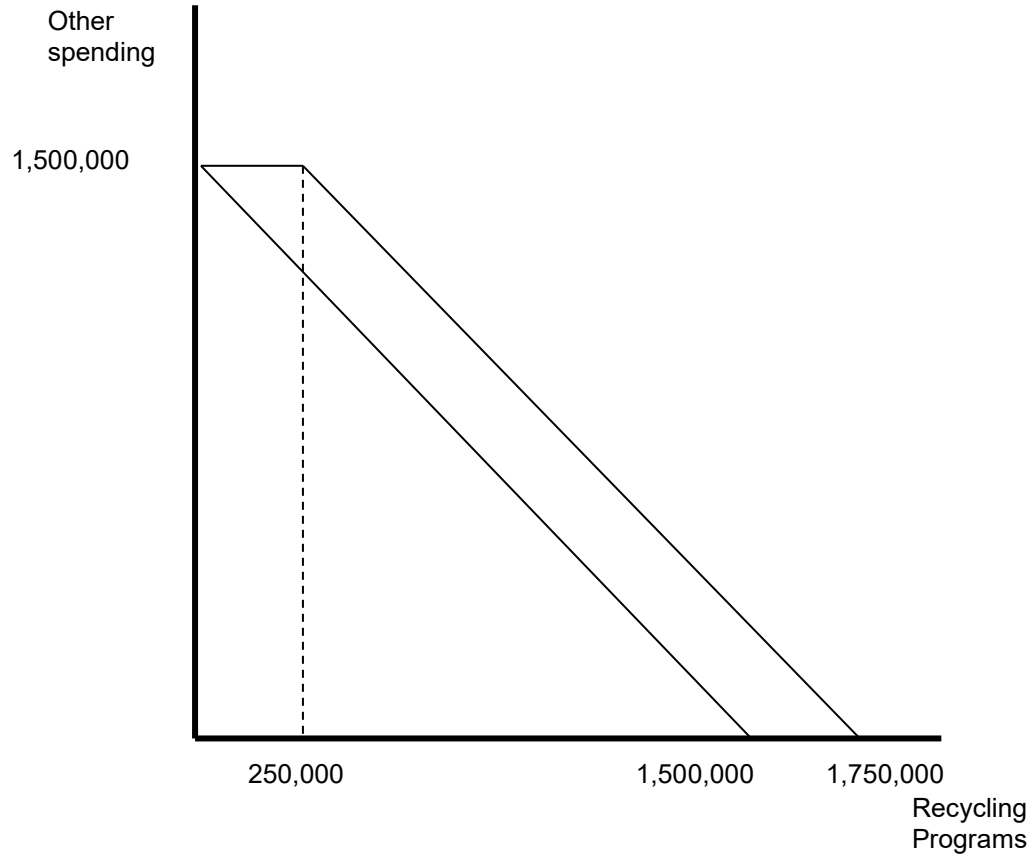


We can see this better by enlarging the area where the indifference curves are:



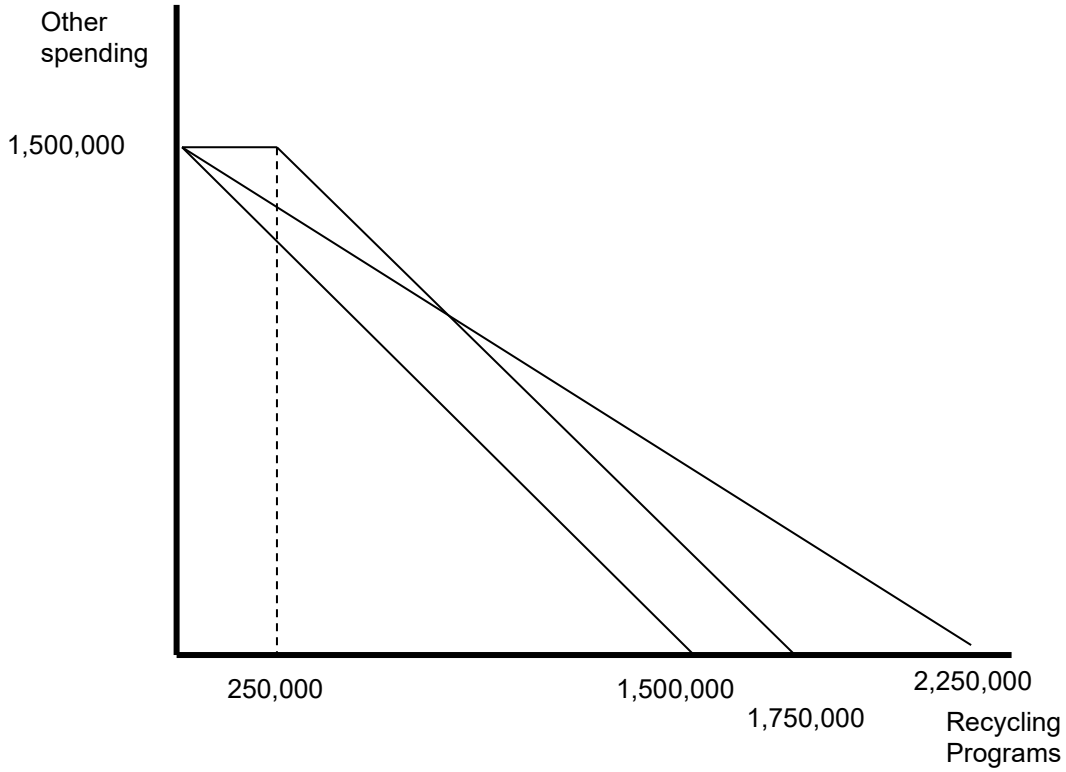
The original budget constraint from part (a) and the budget constraint at the new prices with the government subsidy both go through point A, since they allow the same consumption as before. Before the price increase, point A was an optimal bundle, so the indifference curve is tangent to the original budget constraint. Since gasoline prices are now higher, the slope of the new budget constraint is steeper. Thus, the indifference curve cannot also be tangent to the new budget line at point A. Instead, *the new budget constraint goes through the indifference curve*. As a result, it is possible to draw a higher indifference curve that is tangent to the new budget constraint.

4. a)



The initial budget constraint allows the community to spend $1,500,000 on recycling or other goods. With the new policy, if this community spends all of its own money on other spending, it still has $250,000 to spend on recycling. Similarly, if it spends all of its money on recycling, it now has $1,7500,000 to spend.

b)



The subsidy lowers the price of recycling. For every dollar spent by the community, the federal government contributes 50 cents. Thus, the price of recycling services under this program is $0.67, since the community pays 2/3 of the cost for every unit of recycling services (= $1/$1.50). Under this policy, the community can afford $2.25 million worth of recycling services (= 1,500,000/0.66666).

c) Communities that prefer little recycling will prefer the block grant. For these communities, the $250,000 of income given with the block grant is more valuable than the subsidy. A community would need to spend at least $500,000 on recycling to get back $250,000 from the government under the subsidy in proposal B.

While I was looking for an intuitive answer, we can see this on the graph above. If the level of spending on recycling is low, the community will be to the left of the intersection of the two budget constraints. In this area, the block grant budget constraint is highest.

d) If a community purchases more than $750,000 on recycling programs, it will get back more than $250,000 from the government. To see this, note that to get $750,000 worth of recycling services, the community spends $500,000 under each proposal. Under proposal A, the $250,000 block grant makes up the difference to get to $750,000, and under proposal B, the 50 cent subsidy provides the additional $250,000. Thus, Representative Pooh's community gets more from the government under the matching grant (proposal B).

Again, while it wasn't what I was looking for in this answer, you can also see this on the graph, as Representative Pooh's community is to the right of the intersection of the two budget constraints. In this area, the matching grant budget constraint is highest.