

**Problem Set #3**  
**PAI 897**  
**Professor David Popp**  
**Fall 2023**

**Solutions Available Wednesday, October 13**

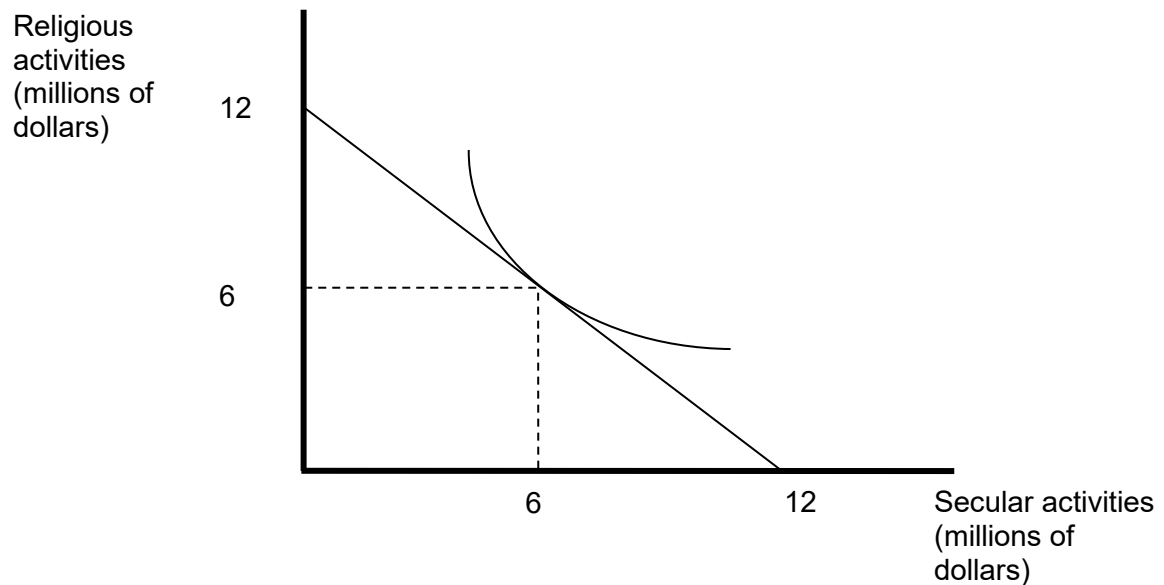
1. This problem is designed to help you practice the concept of utility maximization. You will need to download the Excel worksheet utilmax.xls from the problems page of the class web site to complete the problem.

The worksheet includes a sheet labeled “calculations.” On the calculations sheet, I have plotted an indifference curve (If you are interested, the curve is derived from the utility function  $U(x,y) = \sqrt{xy}$ , although you do not need to know that to complete the problem.)

Given the initial income and prices ( $I=\$20$ ,  $P_x=\$1$ ,  $P_y=\$1$ ,  $X=15$ ), the allocation of  $x$  and  $y$  is not optimal. Your task is to find the combination that does maximize utility. You may change any of the boxes that are highlighted on the calculations sheet (income,  $P_x$ ,  $P_y$ , or  $X$ ). You cannot change the consumption of  $y$ . This is found by spending all the remaining income. As you change variables, observe what happens to consumption of good  $y$ , marginal utility per dollar of each good, and total utility. Also, observe what happens to the indifference curve diagram. Included below are some questions to guide your use of this sheet.

- a) Given the initial setup ( $I=\$20$ ,  $P_x=\$1$ ,  $P_y=\$1$ ,  $X=15$ ), utility is not maximized. How can we tell this by looking at marginal utility per dollar? How can we tell this by looking at the graph?
- b) Given the relationship between the marginal utility per dollar of  $x$  and  $y$ , how should you change the level of  $x$  to improve utility? Why?
- c) Try improving utility by changing  $x$  in the direction you state in question 2. Were you correct? If you were, total utility should increase. If you weren't, try moving  $x$  in the other direction (and be sure that you understand why you were wrong – if you don't, please ask me!)
- d) What happened to the marginal utility per dollar of  $x$  as you went from (a) to (c)? Why?
- e) What happened to the marginal utility per dollar of  $y$  as you went from (a) to (c)? Why?
- f) Keep changing the value of  $x$  until a maximum is reached. How do you know when this has occurred?
- g) (optional) Feel free to try different scenarios by changing the price or income, as well as the amount of  $x$  that is consumed.

2. The federal government wants to support education but must not support religion. To this end, it gives the University of Notre Dame \$2 million with the stipulation that this money be used for secular (that is, not overtly religious) purposes only. The graph below shows Notre Dame's pre-federal-gift budget constraint and utility.



- Reproduce the above diagram. On your diagram, show how the federal grant changes Notre Dame's budget constraint.
  - Given the indifference curve drawn on the graph, would the university's welfare change if the grant came without the secular-use restriction? Why or why not?
3. Suppose that, in January, the typical low-income consumer had \$1000 of monthly income, and spent this money on gasoline (G) and other goods (O). At the time, the price of gasoline was \$2 per gallon. At those prices, the typical low-income consumer spent \$200 per month on gasoline, purchasing 100 gallons.
- Sketch this initial condition using a budget constraint and indifference curve. Explain intuitively what is true at the point where the consumer maximizes his utility (that is, why is utility maximized – not just “because it's where the curve is highest.”).
  - Since then, gasoline prices have risen dramatically, to a new price of \$3 per gallon. After the price increase, the typical low income consumer now purchases 85 gallons of gasoline per month. Redraw the above graph, and illustrate the effect of the price change on the graph.
  - Concerned over the effect of high gas prices on low-income consumers, the government decides to give each low-income consumer enough cash so that they can purchase the same bundle of goods that they purchased before prices increased. Redraw your graph from part (b). Add a new budget constraint to show how much income must be given back. How will the utility of consumers after the rebate compare to the utility they had in January? Explain intuitively what has happened.

4. To encourage communities to spend more on recycling programs, Representative C. Robin has proposed providing federal aid to communities to help pay for these programs. There are two proposals:

*Proposal A – block grants:* This proposal would give each community a block grant of \$250,000. The money could only be spent on recycling programs.

*Proposal B – matching grants:* This proposal would subsidize spending on recycling programs. For each dollar spend on recycling programs by local communities, the federal government would provide \$0.50 of aid.

- a) Suppose that a typical community has a budget of \$1,500,000. Draw a budget constraint for such a community showing their current choices (pre-policy). Then, add a second budget constraint depicting the community's options under proposal A. Be sure to clearly label all endpoints of each budget constraint.
- b) Please reproduce your figure from part (a). Then, add a third budget constraint depicting the community's options under proposal B. Once again, please clearly label all endpoints.
- c) Representative Robin is concerned about communities that currently spend little on recycling programs. Which of the proposals above will these communities prefer? Please explain the intuition of your answer in a way that Representative Robin, who has little formal training in economics, would understand.
- d) Representative W. Pooh represents a community that spends more than \$750,000 on recycling programs devices. Which proposal makes Representative Pooh's community better off? Why? Again, please explain your answer in a way that would be clear to a non-economist. (*Hint:* Calculate how much money is left for other spending under each proposal when a community spends a total of \$750,000 on recycling programs.)