PAI 897 **Professor David Popp**
Solutions to Problem Set #2 **Fall 2023**

1. a) The formula for elasticity is:

$$\varepsilon = \frac{\%\Delta Q}{\%\Delta P}$$

Recall that $\%\Delta Q = \Delta Q/Q$ and $\%\Delta P = \Delta P/P$. Thus, the percentage change in quantity = -600/2000 = -0.3, and the percentage change in price is 2/8 = 0.25. From this, we calculate the elasticity to be -1.2:

$$\epsilon = \frac{-0.3}{0.25} = -1.2$$

b) Revenue will decrease. Demand is elastic. Thus, the resulting decrease in quantity will more than offset the revenues gained from those who still come to the museum.

Two common errors here were:

1. Using the new quantity and price, rather than the original quantity and price, to calculate the percentage change, and
2. Using 10% for the change in price. While the proposed price increase is 10%, you must use the previously observed data to calculate the elasticity. The change in quantity that occurred previously was the result of a 25% price increase. Thus, you must use 25%, not 10%, as the change in price.

Note that simply calculating the new and old revenues would only receive partial credit. Simply calculating these revenues shows that revenues increase, but does not explain why they increased.

2. a) For this problem, we simply use the elasticity formula:

$$\varepsilon_D = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q/Q_0}{\Delta P/P_0} = \frac{\Delta Q}{\Delta P}\frac{P_0}{Q_0}$$

We know the original price and quantity, as well as the proposed price change (-$2). With that information, we can plug in what we know to the elasticity formula and solve for ΔQ:

$$\Delta Q = \frac{\varepsilon_D(Q_0)(\Delta P)}{P_0} = \frac{-1.25(960)(-2)}{12} = 200$$

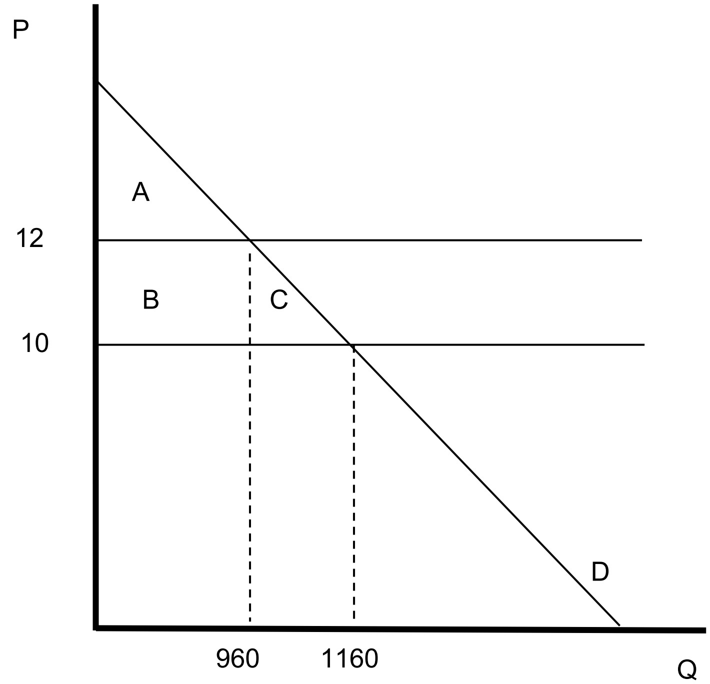
There will be 200 additional visitors, for a total of **1,160** visitors.

If you used the percentage change version of the formula, the calculation is as follows. Begin by calculating $\%\Delta P$, which equals $\Delta P/P_0$. Thus, the percentage change in price = -2/12 = -0.1667. Since we know the elasticity, we can plug this into the elasticity formula:

$$-1.25 = \frac{\%\Delta Q}{-0.1667}$$
$$\%\Delta Q = (-1.25)(-0.1667) = 0.20833$$

There will be 20.833% more visits. To find the number of trips, we need to find how many more trips will be taken after the fee increase. This equals 960 x 0.20833 = 200. The number of trips increases by 200, so that 1,160 total visitors come to the park after the entrance fee falls.

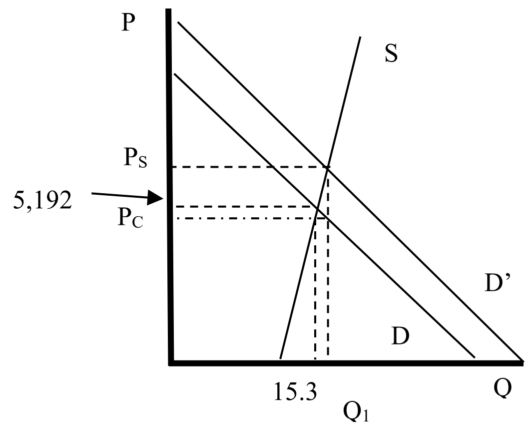
b) Note that you don't need to know the end points of the demand curve to find the lost consumer surplus, nor do you need the supply curve. Some of you chose to add a supply curve. That is fine, but it doesn't change what is needed to calculate the change in consumer surplus, as what matters is the area between the two prices. Also note that we are *moving along* the demand curve. Demand does not shift. The illustration below shows what we are looking for:



With the original entrance fee of $12, consumer surplus is just area A. When the fee falls to $10, consumer surplus is areas A, B, and C. Thus, we can find the lost consumer surplus by calculating areas B & C. Even though we don't know the *y*-intercept of the demand curve, we do know enough to calculate this area.

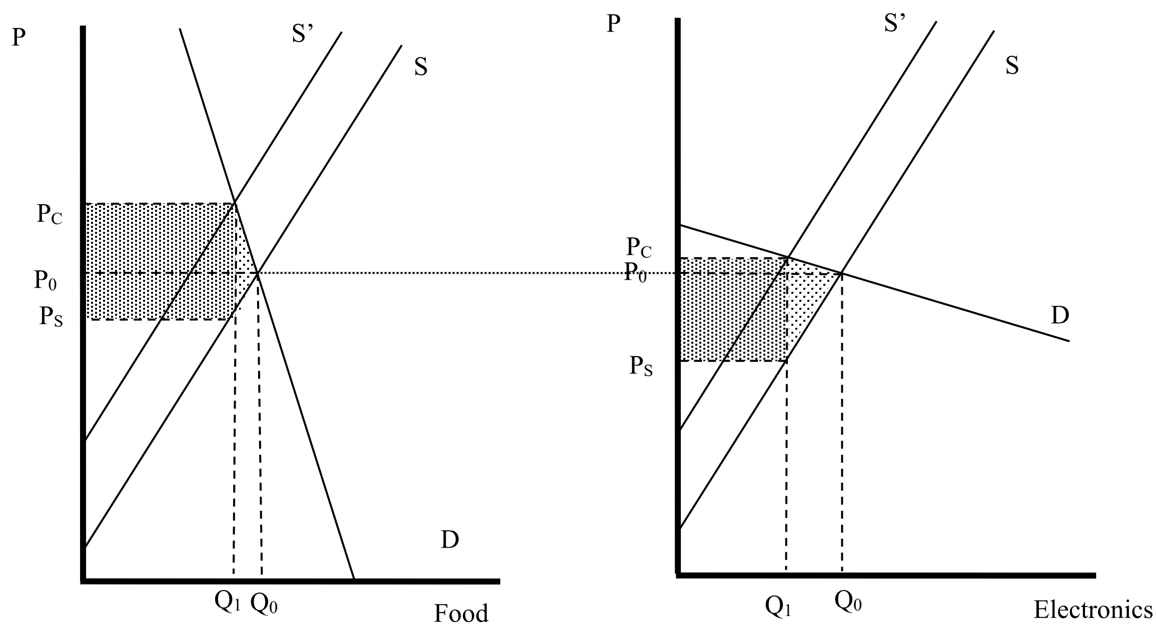
- B is a rectangle with a height of 2 and a length of 960. Its area is 1,920.
- C is a triangle with a height of 2 and a base of 200 (= 1,160-960). Its area is 0.5(2)(200) = 200.
- Thus, the increased consumer surplus = 1,920 + 200 = **$2,120**.

3. Colleges, not students, will be the main beneficiaries. The assumption that colleges are operating near capacity means that supply is nearly fixed – it is inelastic. Just as inelastic parties bear the burden of taxation, they reap the benefits of tax credits, as shown in the graph below.



The tax credit shifts the demand for college up. However, since quantity cannot increase by much, tuitions received by colleges increase (P_S). Students pay P_C, which equals P_S minus the tax credit. Note that although the price students pay does fall somewhat, the drop is not nearly as large as the gain received by colleges. Most of the $1,000 from the tax credit goes to colleges. If colleges were not at full capacity, students would benefit more, as enrollments could increase without tuition increasing as much.

4. I use the following diagram to answer each of the questions. Note that since demand for basic food is inelastic, it has a steep demand curve, and since electronics are luxuries, their demand is more elastic. Thus, the demand curve is flatter. Since the problem says nothing about the supply curves, I assume that they are the same in each case. This makes the problem simpler. Since the supply curve is the same for each, shifting the supply curve is the best way to represent the tax and make comparisons. Since the tax is to be the same size, the supply curve shifts by the same amount in each diagram.



a) Consumer prices rise more with the tax on food. This is because demand for food is inelastic. Thus, it is harder for consumers to avoid the tax. Because demand for electronics is elastic, the quantity purchased falls more with a tax on electronics. Thus suppliers, rather than consumers, bear the bigger burden of the electronic tax.

b) The tax on food will raise more revenue. Recall that when prices are raised for inelastic goods, revenue increases, and when prices are raised for elastic goods, revenue decreases. Because there is a smaller change in quantity with the tax on food, there are more sales after the tax, so more revenue is raised. This is shown on the graph by the dark shaded rectangle on each graph, which is larger in the case of food.

c) The tax on electronics has a greater deadweight loss. Deadweight loss represents the lost opportunity from sales that should have occurred, but do not after the tax is in place. Since there are more lost transactions for electronics (because demand is more elastic), the tax on electronics has a greater deadweight loss. On the graph, this is represented by the lightly shaded triangles, which are larger for electronics.

d) There is no right answer to this question. The purpose was to get you to think about different approaches to a problem and their implications. This question is an example of a *normative* problem, and your response depends on which goals are most important. In this case, the tax on food is more effective for raising revenue. It is also more efficient – there is less deadweight loss, because the change in equilibrium quantity is smaller. However, opponents of the tax on food raise a valid point when they note that it is not fair. These foods are basic necessities, so it is hard for consumers to avoid the tax. Moreover, lower income people will be hurt by the food tax more, as they spend a larger portion of their income on food. In contrast, rich people will hardly notice, since they spend a smaller percentage of their income on milk (Bill Gates may have thousands times more money than you or I, but he doesn't drink thousands of times more milk!). On the other hand, electronics are a luxury, and thus consumed more by those with higher incomes. These are the types of issues I hoped you would think about in this part of the question.

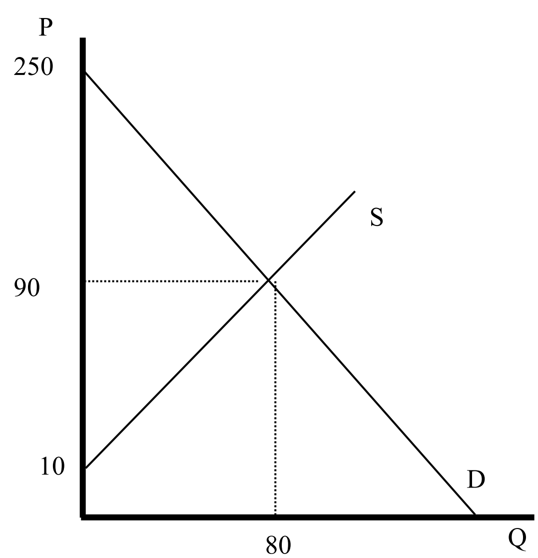
5. a) The equilibrium price and quantity are:

$$250 - 2Q = 10 + Q$$
$$240 = 3Q$$
$$Q = 240/3$$
$$\mathbf{Q = 80}$$

Next, we plug this answer into either the supply or demand equation to find the equilibrium price:

P = 250 - 2(80)		P = 10 + 80
P = 250 – 160	*or*	
P = $90		**P = $90**

b)

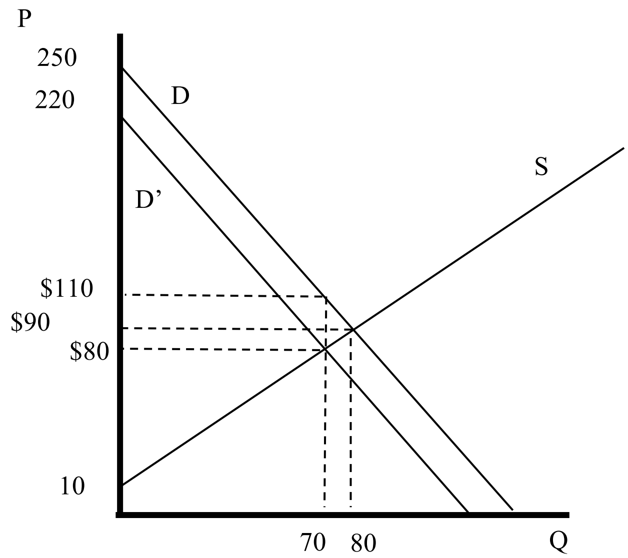


Consumer surplus is the triangle above the price and below demand. It has a height of 160 (= 250 – 90) and a base of 80. Its area = 0.5(160)(80) = **$6,400**.

Producer surplus is the triangle below price and above supply. It has a height of 80 (= 90 – 10) and a base of 80. Its area = 0.5(80)(80) = **$3,200**.

c) The result of the tax is to shift either the supply curve or demand curve in. Since the legal incidence of the tax is on consumers, we shift the demand curve.

The demand curve shifts in by the amount of the tax. The new demand curve represents the demand curve faced by suppliers. To graph it, reduce the y-intercept of the demand equation by the amount of the tax: $30.



Algebraically, recall that $P_S = P_C$-tax. That is, what consumers pay is split between suppliers and the tax. The demand curve becomes $P_C = 250 – 2Q – 30 = 220$ - Q. Graphically, note that the y-intercept of the graph has shifted up by the amount of the tax.

We begin by finding the new equilibrium. Equate the new demand curve with the old supply curve.

$$220 – 2Q = 10 + Q$$
$$210 = 3Q$$
$$Q = 210/3$$
$$\mathbf{Q = 70}$$

We plug this quantity into the *original* supply curve to get the price suppliers keep:

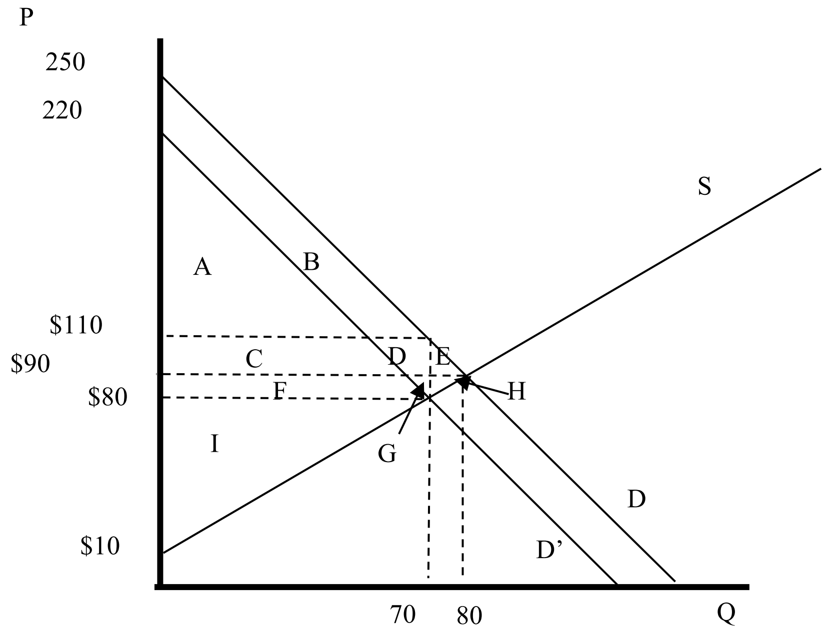
$$P_S = 10 + 70 = \mathbf{\$80}$$

Similarly, we plug the new quantity into the *original* demand curve to get the price consumers must pay:

$$P_C = 250– 2(70) = \mathbf{\$110}$$

To check our work, note that the difference between these two prices is $30, which is the amount of the tax.

d)



Note that we use the original supply and demand, at the new prices and quantities, to find consumer and producer surplus.

Areas A & B in the above graph represents consumer surplus. This is a triangle with a height of 140 (= 250-110) and a base of 70. Its area = 0.5(140)(70) = **$4,900**.

Area I in the above graph represents producer surplus. This is a triangle with a height of 70 (= 80-10) and a base of 70. Its area = 0.5(70)(70) = **$2,450**.

e) Revenue is simply the tax times the quantity sold.

$30 x 70 = $2100.

On the above graph, revenue is the rectangle represented by areas CDFG.

f) Before the tax, the sum of consumer and producer surplus was $9,600. Afterwards, the sum of consumer surplus, producer surplus, and revenue is $9,450. The difference is $150. Graphically, this is the area of triangles E & H.

This difference is the deadweight loss. It is the value of lost opportunities, because some potentially beneficial transactions do not occur after the tax. For the quantities between 70 and 80, demand is above supply. This tells us that consumers are willing to pay more than the marginal cost of producing the good. However, because of the tax, these units are not sold. The potential producer or consumer lost because of this is the deadweight loss.