

PAI 897

**Solutions to Practice Problems for Quiz #2**  
**Professor David Popp**

Fall 2023

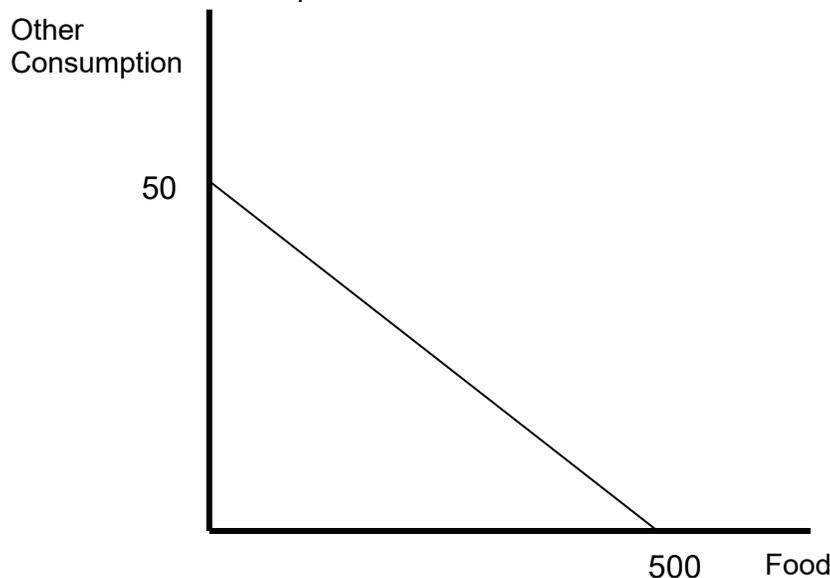
1. After graduation, you are hired by a local advocacy group that supports low-income families. You have been asked to provide analysis of two proposals to provide food aid to low income families.

*Proposal 1* would provide each family with \$100 of food stamps per month. These could only be used to purchase food.

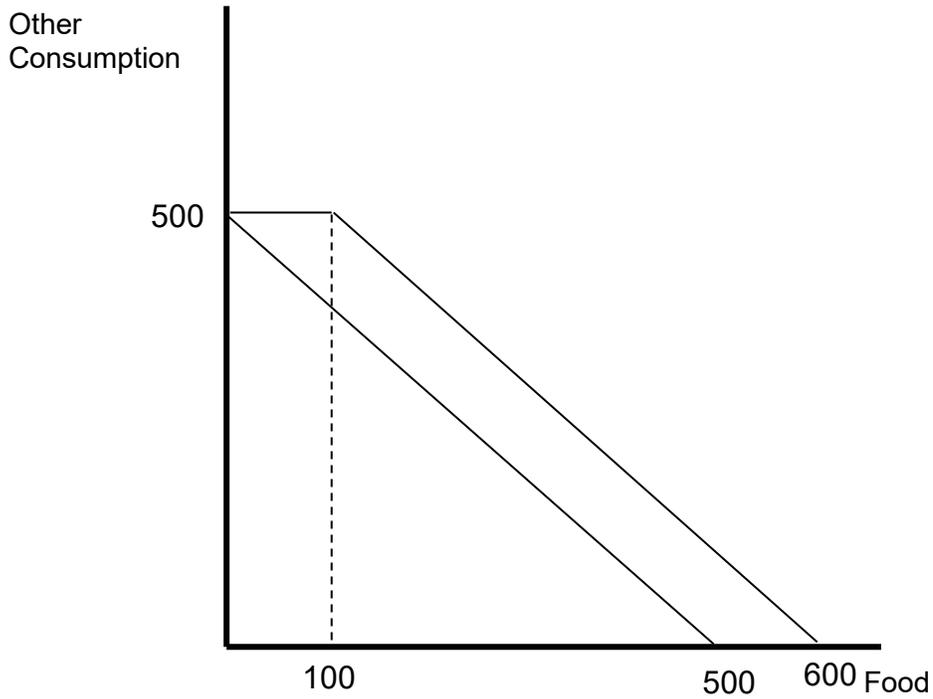
*Proposal 2* would subsidize the purchase of food by these families. For each dollar spent on food, the government would provide a dollar of aid, so that the effective price of food to these families is \$0.50.

You have been asked to analyze the effect of this proposal on a typical low-income family with an income of \$500 per month.

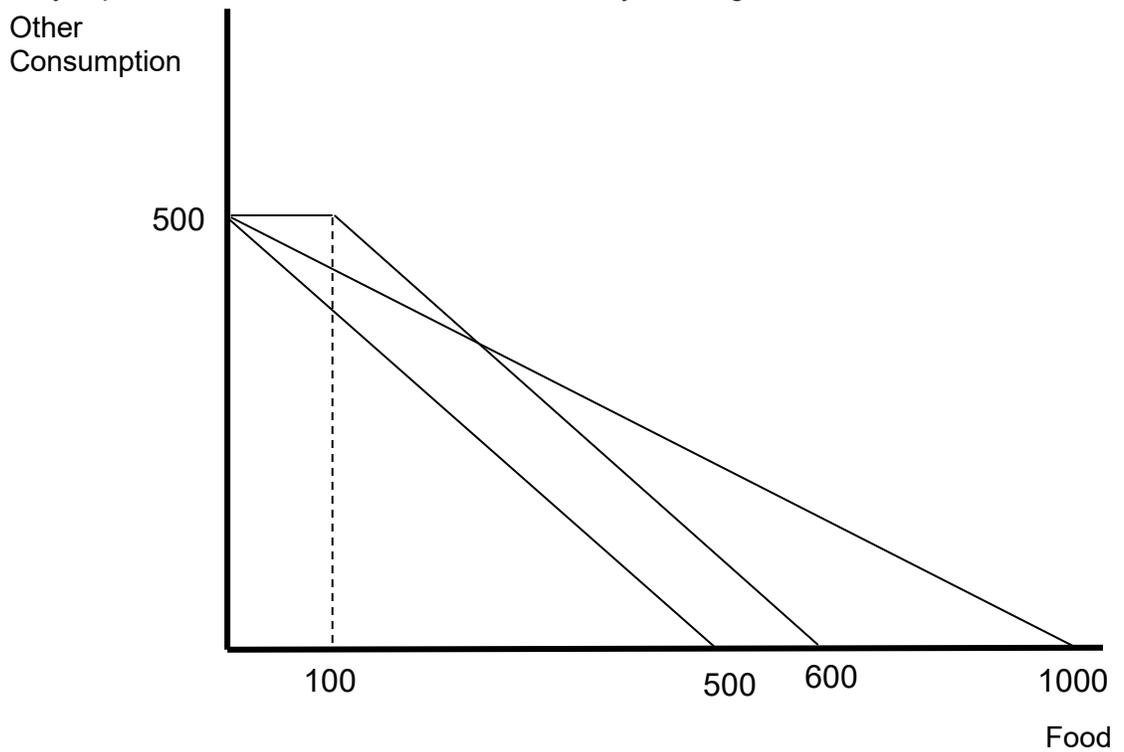
- a) Draw a budget constraint for the typical family before aid is provided. Place food on the x-axis, and other consumption on the y-axis.
  - b) Reproduce your diagram from part (a). Now, add a budget constraint for proposal 1 (the food stamps).
  - c) Reproduce your diagram to (b). Add a budget constraint representing proposal 2 (the food subsidy).
  - d) Your analysis shows that the typical family buys \$200 worth of food (based on pre-policy prices) when given the food subsidy (proposal 2). Given this, under which proposal is the family better off? Which proposal encourages more food consumption? Explain. Please use your graph from part (c) to help explain your answer. (*Hint: Calculate how much it costs the family to buy \$200 worth of food under each plan.*)
- a) Without any aid, a typical low-income family can spend \$500 on other consumption, purchase \$500 worth of food, or end up somewhere in between.



b) Now, this family can purchase up to \$600 worth of food, and can purchase \$100 worth of food even if they consume \$500 of other goods.

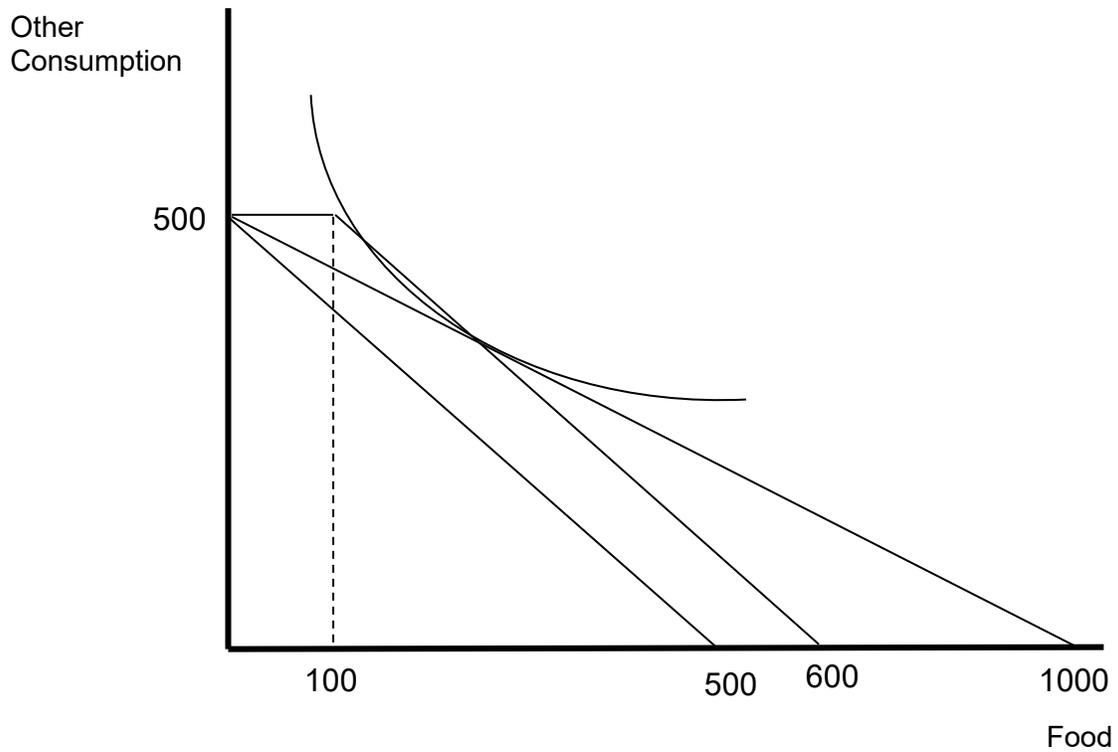


c) The food subsidy lowers the price of food. Thus, the budget constraint rotates. If a family spends all their income on food, they now get \$1,000 worth of food.

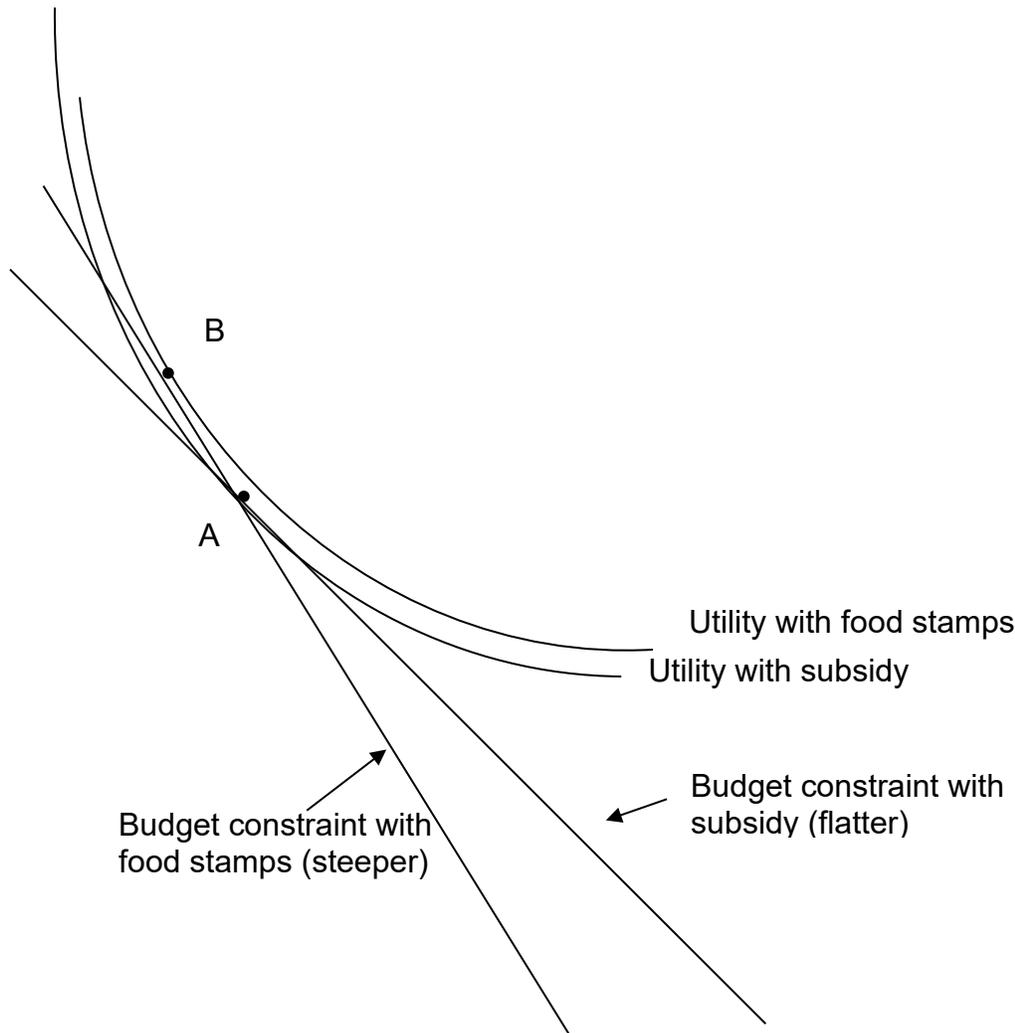


- d) The key to this problem is that the budget constraints intersect at \$200 worth of food. Under the food stamp plan, consumers get \$100 worth of food free, and purchase an additional \$100 worth for \$1 each. Thus, they spend \$100 to get \$200 worth of food. Under the subsidy, the family can also purchase \$200 worth of food for \$100, since each unit of food only costs the consumer \$0.50.

Given this, notice that if the family is maximizing utility under the subsidy at this intersection, then the budget constraint for the food stamp program goes through their indifference curve.



We can see this better by enlarging the section of the graph where the lines intersect:



The budget constraints for the two policies intersect at \$200 worth of food, which is also where the indifference curve is tangent to the subsidy budget constraint. Since the slope of the food stamp budget constraint is steeper, the indifference curve cannot also be tangent to the food stamp line there. Instead, *the budget constraint for the food stamp plan goes through the indifference curve*. As a result, it is possible to draw a higher indifference curve that is tangent to the food stamp budget constraint. Consumers are better off with food stamps.

However, while consumers are happier with food stamps, they will consume less food. Note that the new optimal bundle (point B under food stamps) is to the left of the optimal bundle under the subsidy (point A). The intuition is that the subsidy leads to greater consumption of food because it lowers the price of food. Thus, it induces families to substitute food for other goods. The key here is that, since the subsidy changes prices, it has both an income effect and a substitution effect. The food stamp policy does not have a substitution effect, as the prices do not change.

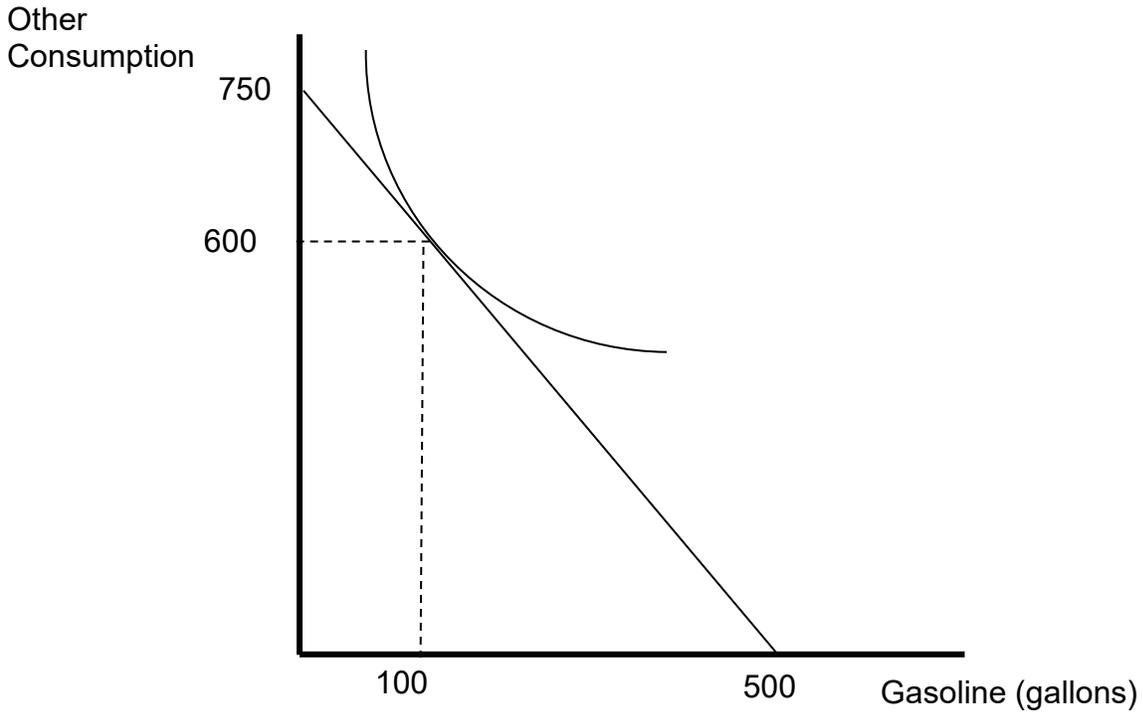
2. The country of Pangaea currently subsidizes gasoline. Concerned over both the environmental impact and the cost of these subsidies, the government is considering eliminating the subsidy. You have been asked to analyze the effect of eliminating the subsidy on both consumers and the government's budget. To begin, consider the following facts for a typical family in Pangaea.

- With the subsidy, gasoline costs \$1.50 per gallon. The typical family purchases 100 gallons of gasoline at this price.
  - The cost of gasoline without any subsidy would be \$2.50 per gallon.
  - The typical family has \$750 of disposable income to spend on gasoline or other consumption goods.
- a) Using an indifference curve and budget constraint, sketch the initial condition with the subsidy. Place other consumption on the  $y$ -axis and gallons of gasoline on the  $x$ -axis. Be sure to show the endpoints of the budget constraint, as well as the levels of gasoline and other consumption chosen by this family.
- b) Reproduce your diagram from part (a). Now, consider what happens when the subsidy is removed and gasoline prices increase to \$2.50 per gallon. Add the new budget constraint to the diagram, along with a new indifference curve showing approximately how much gasoline the family will consume when prices are higher.
- c) Reproduce your answer to (b). To compensate families for higher prices, the government uses the money saved from eliminating the gasoline subsidy to lower income taxes. Suppose that the amount of extra income each family gets is just enough to return their utility to what it was when gasoline was subsidized. Add a budget constraint representing this policy to your diagram.

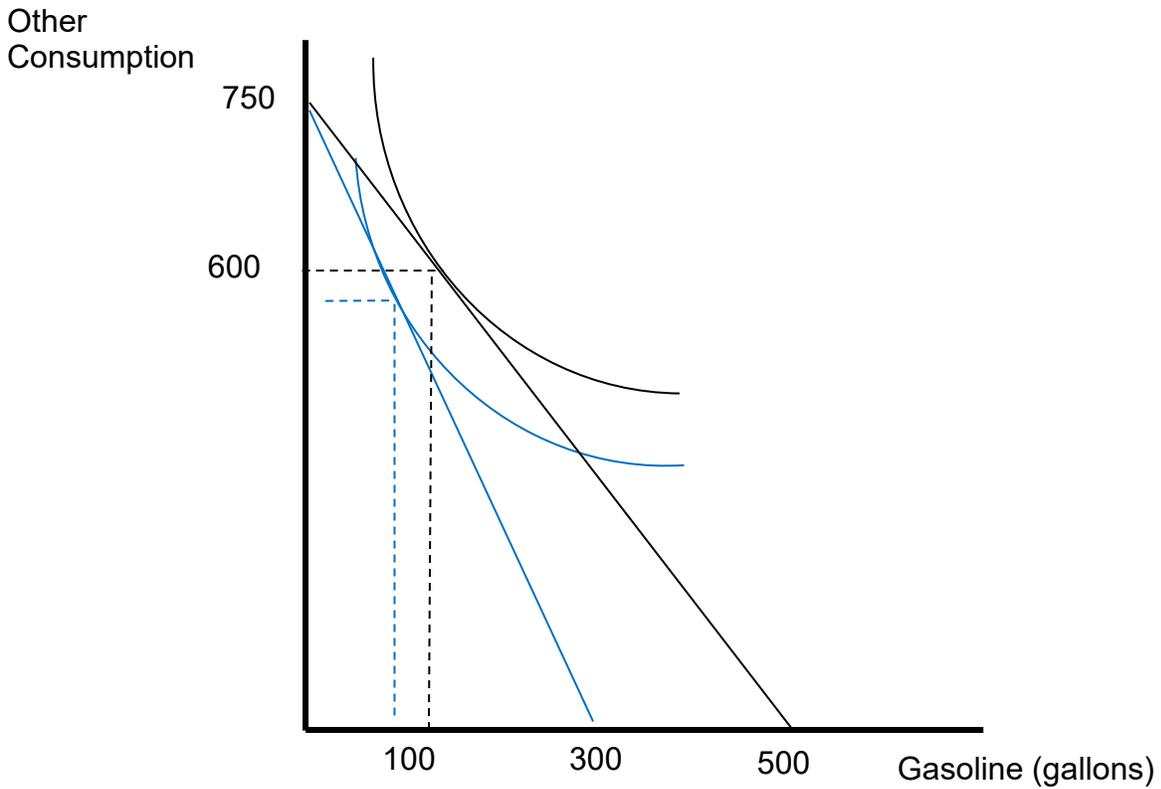
After taxes are reduced, will families choose to purchase 100 gallons of gasoline, as they did under the subsidy? Explain briefly.

- d) Finally, consider the effects of lowering taxes on government revenue. Compared to what they spent subsidizing gasoline, will the income tax cut cost the government more, less, or exactly the same amount? How do you know this?

- a) To draw the budget constraint, note that consumers can buy up to 500 gallons of gasoline (=  $\$750/\$1.5$ ) or \$750 worth of other goods. Note that these endpoints are what we need for the budget constraint – we want to show *what is possible*, not just what the consumers actually do. A typical family actually chooses 100 gallons of gas. Since each gallon costs \$1.5, this leaves them \$600 to spend on other consumption. This is shown by drawing an indifference curve tangent to the budget constraint at 100 gallons of gasoline. This is the highest possible indifference curve given the budget constraint.



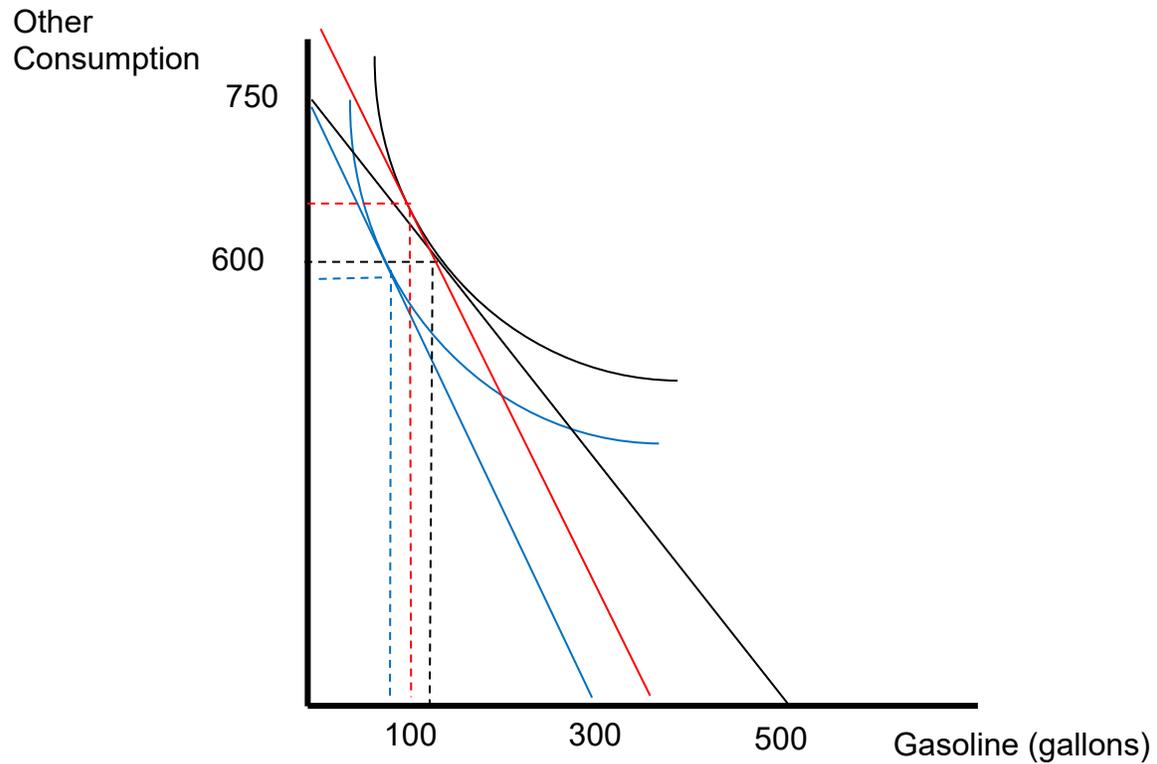
- b) The price increase rotates the budget constraint in, as shown in blue below. If this family spent all their income on gasoline, they could only afford 300 gallons.



While we don't know exactly how much gasoline this family will now consume, we do know that the indifference curve must shift down to be tangent with the new blue budget constraint. Thus, we expect both consumption of gasoline and other goods to fall. This new indifference curve, tangent to the new budget constraint, is lower than the original indifference curve. Utility has fallen.

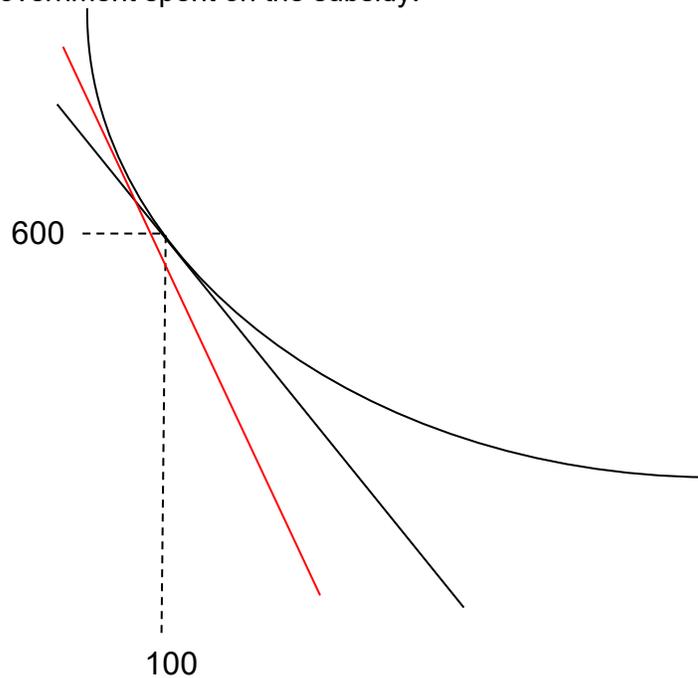
- c) This budget constraint is shown in red above. The budget constraint is parallel to the blue constraint, since the price of gasoline remains \$2.50, but shifts out until it is tangent to the original black indifference curve. Thus, consumers now have the same utility as before.

While we are not told exactly how much gasoline consumers choose at this point, note that it is less than 100 gallons. That is because of the *substitution effect*. Even though consumers have enough income to restore their original utility, they choose to buy less gasoline because it is more expensive. By giving the consumers enough income to restore their initial utility, we have removed the *income effect* of the price change, leaving only the substitution effect remaining.



- d) Note that the new budget constraint does not give this family enough income to purchase what they did before. While the family will spend some of the extra income on gasoline, they will spend more of it on other consumption than they would have had the cost of fuel been \$1.50 per gallon. As a result, the government does not need to give this family enough income to get all the way back to their original consumption point. Thus, the amount of money given back in lower taxes must be *less* than the original cost of the subsidy.

The above intuition is all that was necessary to answer this question correctly. To see the response graphically, note that the red budget line is steeper than the original black budget line. Thus, it is tangent to the black indifference curve at a higher point on the indifference curve, as shown below. While the family will spend some of the extra income on gasoline, they will spend more of it on other consumption than they would have had the cost of fuel been \$1.50 per gallon. Because it is below the original consumption point, this represents less income than the government spent on the subsidy.



A common mistake here is simply saying that the income tax cut cost more because not everyone consumes gas. The question asks you to analyze this policy for a typical family – the typical family does consume gasoline. I'm looking for you to compare the magnitude of two *comparable* policies. Adding extra information like that complicates the analysis unnecessarily. If families who don't buy gasoline receive a tax cut, they are clearly better off, which is not the goal of the policy in question (c). Realistically, if we want to consider multiple types of families, we can also adjust the tax cut amount so that the government doesn't end up spending more after removing the subsidy.

3. You are the manager of Meals on Wheels, a non-profit organization that delivers prepared meals to elderly citizens. Five new volunteers have joined your organization, and it is your job to assign each volunteer to a neighborhood where they will make deliveries. Your organization serves two areas: Uptown and Downtown. Based on the distances between homes, traffic volumes, and the number of people served, you have calculated the following data for the productivity of volunteers in each neighborhood:

<i>Uptown</i>				<i>Downtown</i>			
# Volunteers	MP	AP	TP	# Volunteers	MP	AP	TP
0	0	0	--	0	0	0	--
1	24	24	24	1	16	16	16
2	20	22	44	2	14	15	30
3	13	19	57	3	12	14	42
4	7	16	64	4	10	13	52
5	1	13	65	5	8	12	60

Consistent with the organization's goals, your responsibility is to allocate the five new volunteers so as to maximize the number of people served. To do this, how many of the volunteers should be assigned to Uptown, and how many should be assigned Downtown? Explain how you know this (without simply saying that you added up all the combinations and found the largest value – I'm looking for some economic intuition here, and you shouldn't need to spend much time on this question!)

To answer this question, you simply need to compare the marginal product of adding each volunteer to one of the two neighborhoods. We want to place volunteers in the five most productive locations. These are the five options where marginal product is highest. This requires **three** volunteers Uptown and **two** volunteers Downtown.

We can see the intuition behind this by finding the best location for each of the five volunteers. For the first volunteer, the marginal product is 24 in Uptown, but only 16 Downtown. Thus, this worker should be assigned Uptown.

Now move to the second volunteer. The marginal product of a second worker Uptown is 20. This is still greater than the marginal product of 16 for a Downtown volunteer, so this second worker should also be assigned Uptown. However, the third volunteer will be more valuable Downtown, as the marginal product of the first volunteer there (16) is greater than the marginal product of a third worker Uptown (14).

Using similar logic, the fourth volunteer should also be assigned Downtown (MP of 14 vs. 13 for Uptown). The fifth volunteer should go Uptown, as the marginal product of 13 is better than the marginal product of 12 for a third worker Downtown.

Thus, the final allocation is **three** workers Uptown and **two** workers Downtown.

4. A rural electricity district owns two power plants – a hydroelectric plant that can generate power at  $2\phi/\text{kWh}$ , and a natural gas plant that can generate power at a cost of  $4\phi/\text{kWh}$ . Currently, the district generates two-thirds of its power using the hydro plant, and the remaining one-third using the natural gas plant. As a result, the natural gas plant is only operated at one-half of its capacity, while the hydroelectric plant is used at its full capacity. The mayor of Smallville, whose town is served by these power plants, argues that the natural gas plant should be used more, rather than the hydro plant. Because of the gas plant's large size, operating the plant at full capacity would allow the fixed costs of building the natural gas plant to be spread over a larger amount of generated electricity, thus lowering prices for consumers in the district. How would you respond to the mayor's argument?

The mayor's mistake is confusing fixed costs and marginal costs. The costs of building the power plants are sunk. Thus, they are not relevant to the decision making process. While increasing use of the natural gas plant would lower average costs, what matters is the marginal cost of generating power from each. Here, we see that the hydropower plant can generate power at  $2\phi/\text{kWh}$ , while it costs  $4\phi/\text{kWh}$  to generate power using the natural gas plant. Given that the marginal costs of hydropower are cheaper, it makes sense to use the hydropower plant first, and then use the more expensive natural gas plant to meet the remaining demand.

Note also that, even if you interpret the costs above as average costs, using the natural gas plant is not correct. In that case, while it may be true that using the gas plant more would lower the average cost of the natural gas plant, the hydro plant would be used less. Thus, the average cost of the hydro plant would go up.

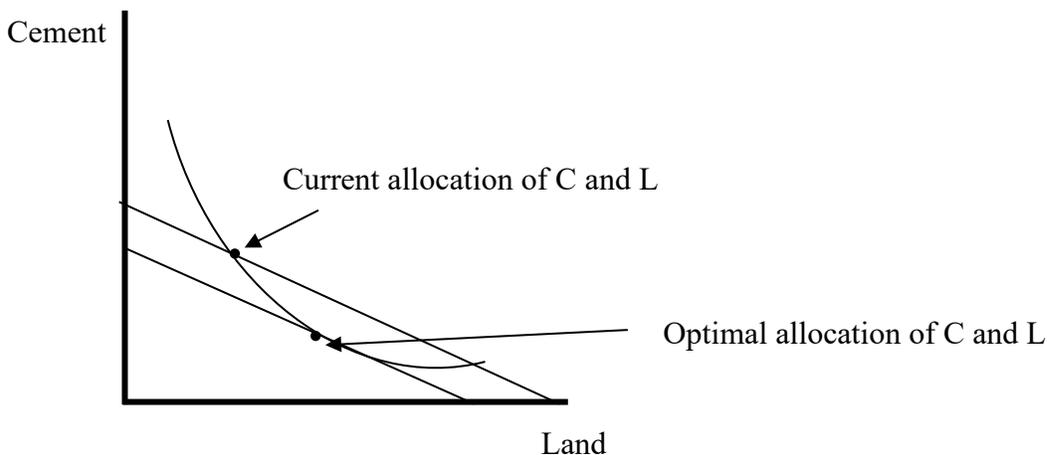
5. The owner of a small retail store does her own accounting work. How would you measure the opportunity cost of her work?

The opportunity cost of her work is the value of her time spent doing accounting. For example, if her time is valued at  $\$20/\text{hour}$ , and she spends 5 hours a week on accounting tasks, the opportunity cost is  $\$100$  per week. Note that the opportunity cost *is not the cost of hiring an accountant*. Rather, to decide whether or not an accountant should be hired, the store owner should compare this opportunity cost to the rates charged by an accountant.

6. Coldwell Banker is employing 10 acres of land and 50 tons of cement to produce 1,000 parking spaces. Land costs \$100 per acre and cement costs \$12 per ton. For the input quantities employed,  $MP_L = 50$  and  $MP_C = 4$ .

- a) Is Coldwell Banker producing parking spaces as cheaply as possible? If so, how do you know this? If not, say what Coldwell Banker could do to improve the situation.  
 b) Illustrate the above scenario using a diagram with an isoquant for 1,000 parking spaces and the appropriate isocost curve.

- a) To minimize costs, Coldwell Banker should operate where  $MP_L/P_L = MP_C/P_C$ . Intuitively, this says that the marginal product per dollar spent on each input is equal. Here, the  $MP_L/P_L = 50/100 = 0.5$ , and  $MP_C/P_C = 4/12 = 0.33$ . Thus, the marginal product per dollar spent on land is greater than the marginal product per dollar spent on cement. Coldwell Banker gets more output from a dollar spent on land than a dollar spent on cement. They should use more land and less cement.



- b) Since Coldwell Banker wants to produce 1000 parking spaces, all the points we consider should be on the same isoquant. The optimal combination is where the isocost line and the isoquant are tangent. Currently, they are using too much cement and not enough land. Thus, they are currently at a point to the left of the optimal point (e.g., a point that uses more cement and less land than the optimal point). Note that the isocost line here is not tangent, and it is higher than the optimal isocost line, since costs are not at the lowest possible level.

7. Because of your excellent economics background, you have been hired as a consultant to evaluate the operations of King Tutelage, a local non-profit organization that tutors low-income school children. To provide services, the group uses two types of workers: professional staff and untrained volunteers. Because they are in such high demand, the professional staff must be paid the going wage rate of \$25/hour for their services. While the volunteers are not paid, there is a cost to hiring them, since they need training and supervision. King Tutelage has found that they typically spend about \$10/hour for the services provided by volunteers.

While professional staff members are more expensive, they are also more productive. Based on the current employment levels, you find that the marginal product of a professional staff member is 5 (that is, 5 children per hour can be helped). In contrast, the marginal product of a volunteer is 1 (that is, only one child per hour can be helped). Given this information, is King Tutelage making the most efficient use of their resources? Have they hired the appropriate number of each type of worker? If so, how did you determine this? If not, what recommendations can you make to improve their efficiency?

Because the volunteers do impose costs on the King Tutelage, we need to consider the productivity of a dollar spent on volunteers versus a dollar spent on professional staff. The marginal product per dollar spent on professional staff is  $0.2 (= 5 \text{ students helped}/\$25)$ . In contrast, volunteers only help 1 student per hour, and cost \$10 per hour for training and supervision. Thus, the marginal product per dollar spent on volunteers is just  $0.1 (= 1/\$10)$ .

Based on these numbers, King Tutelage is not using enough professional staff. A dollar spent on professional staff currently helps twice as many students as a dollar spent on volunteers. They should hire more professional staff and fewer volunteers.

Note that we *cannot conclude* that they should only use professional staff. That is because the marginal products may change as the number of workers change. For examples, additional professionals might not be quite as productive, since presumably the most qualified workers were hired first. Similarly, the remaining volunteers may be more productive, and it may be easier to supervise a smaller number of volunteers. King Tutelage should re-evaluate their situation after hiring some more professional staff to see if the adjustments they have made are sufficient.

8. Traveling Physicians is an international NGO providing health care services to remote locations in low-income countries. Depending on the location, they can either deliver services in-person (such as at a walk-in clinic) or remotely by consulting with patients and local health care providers on-line. You are given the following information on the costs and effectiveness of each type of service provision:

*On-line consultations:* On-line consultations are relatively cheap, costing just \$10 per hour.

However, they are not as effective as services provided in person. Research shows that one hour of on-line consultations treats 5 patients per hour.

*In-person services:* While in-person services are generally more effective, the usefulness of traveling to a clinic varies depending on the quality of the facilities available and the size of the nearby population. Moreover, the costs of traveling to a clinic vary depending on the location. Below are data on in-person services for four local communities:

Community	Cost per hour	Patients treated per hour
Forest Grove	\$50	20
Pride Rock	\$50	40
Savannah Run	\$75	30
River Junction	\$100	60

Traveling Physicians has sufficient funding to provide services in each of these four cities. However, they need to decide whether it is more effective to provide services in-person or on-line. Based on the data given above, in which communities should Traveling Physicians provide services in-person, rather than on-line? Why?

Our choice for each community should depend on which type of service provision treats the most people per dollar spent. For this, we need to consider both the costs and the number of patients treated. We can use MP/P for this evaluation.

The marginal product of an hour of on-line consultation is 5. Each hour costs \$10. Thus,  $MP_{\text{on-line}}/P_{\text{on-line}} = 5/10 = 0.5$ . We can then do the same calculation for in-person services in each community. If the marginal product per dollar for in-person services is greater than 0.5, we should use in-person services in that community. Here are the calculations:

Forest Grove:  $20/50 = 0.4$   
 Pride Rock:  $40/50 = 0.8$   
 Savannah Run:  $30/75 = 0.4$   
 River Junction:  $60/100 = 0.6$

Based on these figures, we should use in-person services in Pride Rock and River Junction, and on-line services in the remaining communities.

Note how both the marginal product and costs are important for this answer. Costs are low in both Forest Grove and Pride Rock. However, it is only worth providing in-person services in Pride Rock, because the marginal product in Forest Grove is also low. It may be that both communities are easy to reach (and thus have low costs), but Forest Grove may also have a lower population, making it not worth the effort to travel there in person.

Note that some students calculated the numbers by flipping marginal product and price. In that case  $P_{\text{on-line}}/MP_{\text{on-line}} = 10/5 = 5$ , and the ratios for each are as follows:

Forest Grove:  $50/20 = 2.5$   
 Pride Rock:  $50/40 = 1.25$   
 Savannah Run:  $75/30 = 2.5$   
 River Junction:  $100/60 = 1.67$

These calculations are fine as long as you interpret them correctly. Instead of patients served per dollar, these figures represent the cost per patient served. Thus, you should choose the *lower* cost in each community. You would still prefer to use in-person services only in Pride Rock and River Junction.

9. To maximize profits, the firm produces at an output level at which total revenue exceeds total costs by the greatest possible amount. At the same time, profits are maximized at the output level at which marginal cost equals marginal revenue. Can you reconcile these two statements?

Profits are maximized when total revenue exceeds total costs by the greatest amount. This occurs when marginal cost equals marginal revenue. The intuition is that the *marginal* costs and revenue tell us the cost and revenue from the *next* unit. If the marginal revenue is greater than the marginal cost, increasing output by one will bring in more money than it will cost to produce the unit. Thus, total profits will increase, and the additional output is worth producing. Similarly, if marginal revenue is less than marginal cost, the next unit will cost more to make than it will bring in. Thus, it is not worth producing. These changes can be made to total profits at any output level *except when marginal revenue and marginal cost are equal*. At that point, the additional revenue brought in by the next unit just covers the cost of producing the next unit, so that total profits remain the same.

10. Consider the following cost data for a perfectly competitive firm:

Q	AVC	MC
6	8.5	8
7	8.571	9
8	8.75	10
9	9	11
10	9.3	12

This company also has fixed costs of \$10.

- a) For the following market prices, find the equilibrium quantity and profits. Be sure to explain how you arrived at your answers:  
 i) \$12, ii) \$10, iii) \$9, iv) \$8.

Because this is a perfectly competitive market, the firm is a price taker. Thus, the firm produces as long as the price is greater than or equal to the marginal cost. If the price falls in between two marginal costs, choose the lower quantity, since the higher one will cost more money than it will make. In each case, profits are total revenue - total cost. Total revenue equals price times quantity sold, and total costs equal  $Q \cdot AVC + FC$ .

- i) With a price of \$12, output is 10. Total revenue equals \$120. Total costs are  $9.3 \cdot 10 + 10 = \$103$ . Profits equal \$17.  
 ii) With a price of \$10, output is 8. Total revenue equals \$80. Total costs are  $8.75 \cdot 8 + 10 = \$80$ . Profits are \$0.  
 iii) With a price of \$9, output is 7. Total revenue equals \$63. Total costs are  $8.571 \cdot 7 + 10 = \$70$ . Profits are \$-7. Note that there are losses here. However, since the losses are less than fixed costs, the business should remain open. Notice that the price is greater than the average variable cost at this point. Intuitively, the firm stays in business because they are covering fixed costs.  
 iv) At a price of \$8, the firm shuts down, because the price is lower than the average variable cost for all quantities. If it did operate, it would be at a quantity of 6. However, at this point, total revenue would be just \$48. Total costs would be  $8.5(6) + 10 = 61$ . Thus, the firm would lose \$13. The firm loses less money by shutting down and just paying the fixed costs.

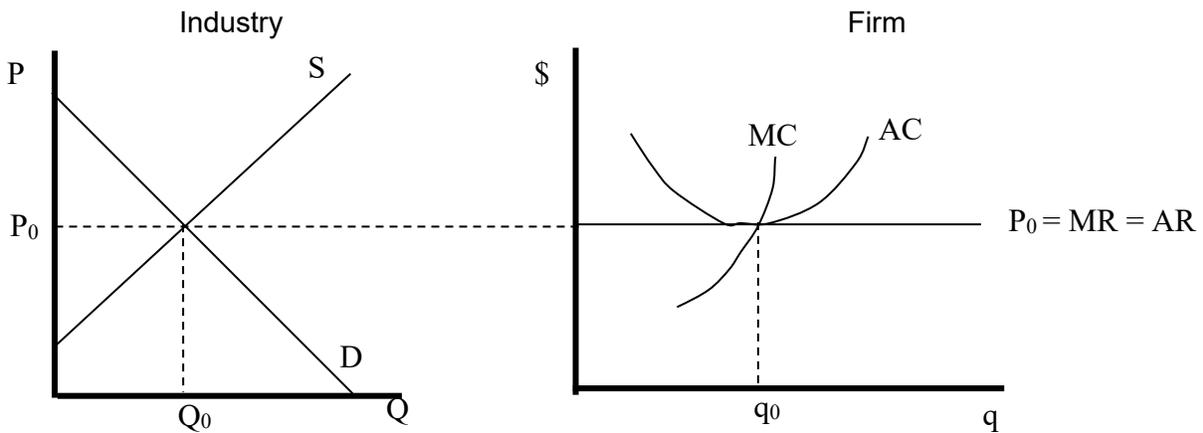
- b) What will be the long-run equilibrium price for this product? Why?

In the long run equilibrium, all firms are making zero profits, and are thus at the minimum of their average total cost curves. In this case, this occurs at a quantity of 8 and a price of \$10. [Note that although the average total cost is also at 10 at a quantity of 7, this occurs because of rounding. If the firm operates at this point, it loses money. The zero profit point, as found in part a, is at 8 units of output.

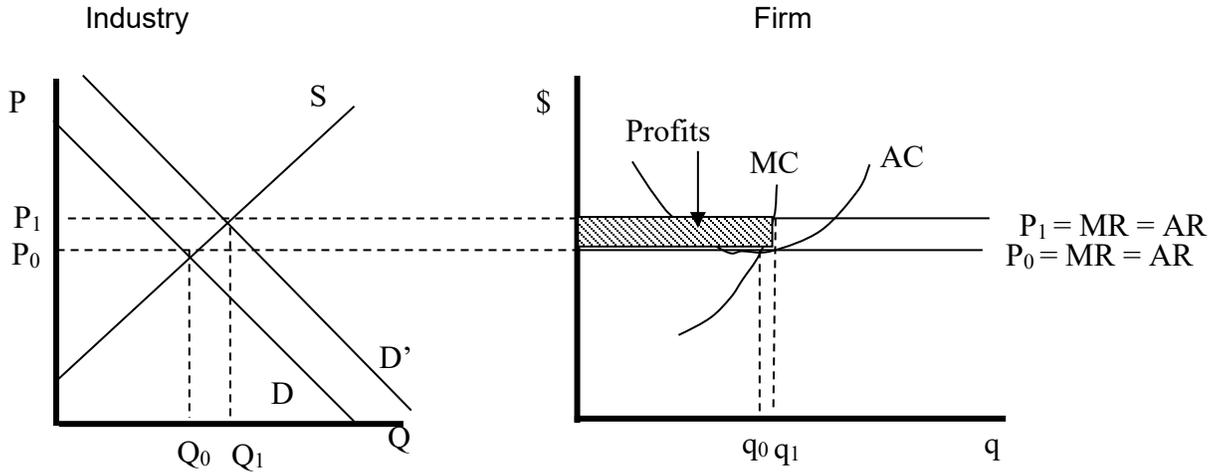
11. Suppose that the market for artificial Christmas trees is initially in a long run equilibrium.

- Draw a supply and demand curve showing the equilibrium price and quantity for artificial Christmas trees. On a second graph, depict a typical tree firm's average and marginal cost curves and depict their profits. Explain why you have drawn this as you did.
- Because of bad weather conditions this year, many pine trees were killed. As a result, fewer natural Christmas trees are available. How will this affect the equilibrium price and quantity for artificial Christmas trees this year? How will this affect profits in the artificial Christmas tree industry? Redraw the diagram of the firm from part a and illustrate the changes you have just described.
- Assuming that the Christmas tree shortage is permanent, what will happen to the artificial tree industry over the next several years? What will the long-run price and quantity of artificial trees be? Redraw your diagrams from parts a and b and depict the changes that occur in the long run.

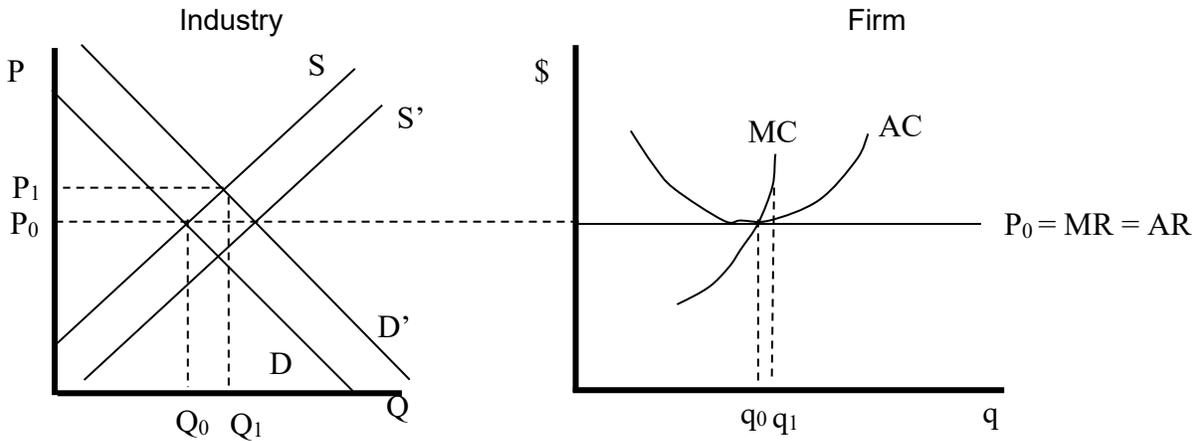
- In long-run equilibrium, firms are making zero economic profits. The price must be equal to the marginal cost at the point where MC intersects the AC curve.



b) Natural Christmas trees are a substitute for artificial trees. Thus, a shortage of natural trees increases demand for artificial trees. This causes the equilibrium price and quantity of artificial trees to increase. Firms will now earn a profit.



c) Because firms are making money, more firms will enter the market. As a result, the supply curve shifts out, lowering the equilibrium price. Firms enter until the price is once again equal to marginal cost at the minimum of AC. That is, long-run equilibrium is restored when firms are again making zero profits.



12. In the village of Pleasantville, residents either work as salaried employees at local offices or are proprietors of stores on Main Street. Salaried employees earn \$30,000 per year. All of the stores are rented to the proprietors by one of several real estate companies who own the buildings. Business is brisk at these stores, so currently the typical store brings in \$70,000 of revenue per year. The typical variable costs needed to run a store in Pleasantville (paying workers, buying material) are \$20,000 year.

- a) What is the opportunity cost of running a store? Explain how you know this.
- b) Given this opportunity cost, what rent will the real estate companies charge? Why?
- c) Suppose that a new highway brings more visitors to town, so that a new store now brings in \$100,000 of revenue per year. What will happen to rents after the increase in revenues? Who will benefit – shop owners or the real estate companies?

a) The opportunity cost of running a store is the value of the next best opportunity. Here, the next best opportunity is working at a salaried job for \$30,000 per year. Thus, the opportunity cost is \$30,000.

b) The stores currently bring in \$70,000 of revenue per year. The variable costs are \$20,000 per year. In addition, the opportunity cost is \$30,000. Thus, storeowners make \$20,000 per year before paying rent. Thus, the rent must equal \$20,000 per year.

To see why, remember that positive profits cannot persist in a competitive market. If the rent were less than \$20,000 per year, more people would want to run stores, since they could do better running a store than working at a salaried job. (Since we've included the opportunity cost of losing a \$30,000 salary as part of the cost of running a store, this is true whenever there are any positive profits for running a store.) More demand for renting stores would drive the rent up until a rent of \$20,000 was reached. Similarly, if the rent were higher than \$20,000 per year, store managers would be losing money, and would leave the business once their leases expired. Demand for stores would fall, causing rents to fall.

c) With a rent of \$20,000, stores will now make \$30,000 profit ( $= \$100,000 - \$20,000 \text{ variable costs} - \$30,000 \text{ opportunity cost} - \$20,000 \text{ rent}$ ). This will increase demand for stores, driving up the rents. The rent will increase until a zero profit condition is reached again. This occurs at a rent of \$50,000. Thus, the entire benefit of the increased traffic goes to the real estate companies that own the stores, rather than to the people that rent the space and run the stores.

13. The city of Danville has just built a new public transportation system, designed as a roller coaster that travels throughout the Tri-State area. They need to decide how much to charge riders of the roller coaster transport system. Because of your background in economics, you have been asked by the city to help set the price.

The fixed costs of operating the system are \$64,000. In addition, the marginal costs of operation are \$8 per rider. Having researched the demand for the roller coaster transport system in Danville, the city has come up with the following table with price, quantity, marginal revenue, and various costs filled in:<sup>1</sup>

P	Q	MR	AC	MC
48	0	48		
44	1000	40	72.00	8
40	2000	32	40.00	8
36	3000	24	29.33	8
32	4000	16	24.00	8
28	5000	8	20.80	8
24	6000	0	18.67	8
20	7000	-8	17.14	8
16	8000	-16	16.00	8
12	9000	-24	15.11	8
8	10000	-32	14.40	8
4	11000	-40	13.82	8
0	12000	-48	13.33	8

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<sup>1</sup> For those that are interested, the table was derived using a demand curve of  $P = 48 - 4Q$  where  $Q$  represents 1,000 riders per day. Note that you do not need to know this to solve the problem.

- a) Phineas, a member of the city council, has taken some economics, and wants to ensure that there is no inefficiency, and thus no deadweight loss, from the pricing scheme. What price will achieve this goal? How many riders will use the roller coaster? How much revenue will these riders generate? What will be the total costs of serving this many riders? Will Danville make money, lose money, or break even on the roller coaster transport system at this price?

To have no deadweight loss, the price should equal marginal cost. This is where the demand curve intersects marginal cost. At this point, all transactions worth at least as much as the cost of providing another ride take place. Since the marginal cost is \$8, this requires setting the price at \$8. From the table, we see that at this price, there will be **10,000** riders.<sup>2</sup>

These riders will generate **\$80,000** of revenue ( $= \$8 \times 10,000$ ).

The total cost of serving these riders is **\$144,000**. You could find this in one of two ways: (1) You could multiply the marginal cost per unit of \$8 by the quantity, and then add the \$64,000 fixed cost ( $= \$8 \times 10,000 + \$64,000$ ). (2) You could simply multiply average costs by quantity ( $= \$14.40 \times 10,000$ ).

Since the total costs are greater than the total revenues, the community loses money. Danville loses **\$64,000** on the roller coaster transport system if it uses marginal cost pricing.

- b) Dr. Doofenshmirtz, a second council member, prefers that Danville take advantage of its market power whenever it can. He asks you to determine the price at which the city, which has a monopoly as the only provider of roller coaster transportation, would maximize its profit from roller coaster riders. What price would that be? How many riders use the roller coaster at that price? Please calculate the total revenue and total costs, as well as the profit for these sales.

To maximize profits, we must find the point where marginal revenue equals marginal costs. Since the marginal cost is \$8, we find the quantity where marginal revenue equals \$8, which occurs when there are **5,000** riders. The price for this quantity is **\$28**.<sup>3</sup>

These riders will generate **\$140,000** of revenue ( $= \$28 \times 5,000$ ).

The total cost of providing these rides is **\$104,000**. You could find this in one of two ways. First, you could multiply the marginal cost per unit of \$8 by the quantity, and then add the \$64,000 fixed cost ( $= \$8 \times 5,000 + \$64,000$ ). Second, you could simply multiply average costs by quantity ( $= \$20.80 \times 5,000$ ).

Finally, we find the profit by subtracting total costs from total revenue. The city makes **\$36,000** of profit at this price.

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<sup>2</sup> Note that you could also get this answer by setting the demand curve equal to marginal cost:  $48 - 4Q = 8$  implies that  $Q = 10$ . However, since I gave you the information in the tables, such calculations were not necessary.

<sup>3</sup> Again, you could use the bisection rule and then find this algebraically. Given the demand curve of  $48 - 4Q$ , we know that  $MR = 48 - 8Q$ . Setting this equal to 8 gives us  $48 - 8Q = 8$ , which simplifies to  $8Q = 40$ , or  $Q = 5$ . We get the price by plugging this quantity into the original demand curve.

- c) A third member of the council, Perry, is concerned about consumers, and does not want the city to maximize its profits. However, it is important to Perry that the city covers its costs, so that Danville does not lose money on its roller coaster venture. Based on the numbers above, what price should the city set to meet Perry's goal? How do you know this?

At this price, how many riders will use the roller coaster? How much revenue will these riders generate? What will be the total costs of serving these riders? What profit, if any, does Danville make from these riders?

Since Perry wants Danville to cover its costs, the profits should equal zero. The trick here is to remember what holds when the profits are equal to zero. When profits equal zero, total costs and total revenue are equal. Thus, *average revenue* and *average cost* are also equal. Since average revenue is just the price, we need to find the price and quantity sold when average costs and price are equal. This is the intuition of average cost pricing that we discussed as a potential policy solution for natural monopolies.

Referring to the table, we see that average costs and price are equal two places: at a price of \$40 or a price of \$16. While I gave students that chose a price of \$40 partial credit for recognizing the  $P=AC$  relationship, given Perry's goals, the better choice is a price of **\$16**. At this price, there will be **8,000** riders. Because Perry is concerned about consumers, it makes more sense to choose the lower price, and thus the ridership. Moreover, choosing a price of \$40 results in a higher price, lower number of riders (2,000) and lower profits than the profit maximizing strategy in part (b). Thus, neither the city nor consumers are better off choosing that price.

These riders will generate **\$128,000** of revenue ( $= \$16 \times 8,000$ ).

The total cost of serving these riders is also **\$128,000**. You could find this in one of two ways. First, you could multiply the marginal cost per unit of \$8 by the quantity, and then add the \$64,000 fixed cost ( $= \$8 \times 8,000 + \$64,000$ ). Second, you could simply multiply average costs by quantity ( $= \$16 \times 8,000$ ).

As expected, Danville makes no profit from the roller coaster transport system in this case. It just breaks even. Thus, it satisfies the goal of serving as many people as possible without losing money.

14. Through the miracles of 19<sup>th</sup> century medicine, Earnest's Extraordinary Elixir has discovered the cure for the common cold. As Earnest is the only one that knows the formula, Earnest has a monopoly on the cold medicine market. The demand for cold medicine is  $P=100 - 5Q$ . The marginal costs of production are equal to \$10. There are no fixed costs.

a) What is Earnest's profit-maximizing output and price?

Profits are maximized where  $MR=MC$ . Since Earnest is a monopolist, his marginal revenue curve bisects the demand curve. Thus,  $MR = 100 - 10Q$ .

$$MR = 100 - 10Q = 10 = MC$$

$$10Q = 90$$

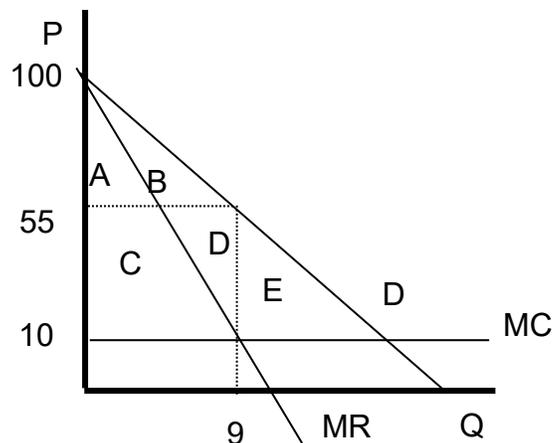
$$Q = 9$$

To get the price, we need to look at the demand curve, to see how much consumers are willing to pay for 9 bottles of medicine. We get:

$$P = 100 - 5(9)$$

$$P = \$55$$

b) What is the producer surplus that corresponds with your answer to part (a)? What is the consumer surplus?



Producer surplus is the area below price and above marginal cost. It is areas C and D on the graph. The area of this is equal to  $45 \times 9 = \$405$ .

Consumer surplus is the area above the price and below demand. It is equal to areas A and B. This area is a triangle with base of 9, and a height of 45 ( $=100-55$ ). Thus, consumer surplus  $= 0.5(9)(45) = \$202.50$ .

- c) Suppose that, over time, others learn the secret to Earnest's Elixir, and the market becomes perfectly competitive. What will the new price and quantity be? Explain how this problem differs from the monopoly problem in part (a).

If competition is allowed, Earnest becomes a price taker, so that  $MR = P$ . In equilibrium, price is equated with MC, which is 10. Thus, the new equilibrium price is **\$10**.

To get Q, we plug into the demand equation to get:

$$10 = 100 - 5Q$$

$$5Q = 90$$

$$\mathbf{Q=18.}$$

- d) Find the new consumer surplus and producer surplus in perfect competition. How does the sum of consumer surplus and producer surplus in the monopoly case [part (b)] compare to consumer surplus with perfect competition? Explain any differences between the two.

Referring back to the graph in part b, the consumer surplus is now areas ABCDE. This is a triangle of base 18, and height 90. Its area is  $0.5(18)(90) = 810$ . Thus, consumer surplus = **\$810**.

There is no producer surplus in this case.

The sum of consumer surplus and profits is \$202.50 greater than the sum from part b. The difference was the deadweight loss from the monopoly (area E). These are beneficial transactions that could have taken place but do not because a monopoly restricts quantity.