PAI 723
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Solutions to Problem Set \#7

1. Since the out-of-town businesses attending events at the convention center would go to other cities if the convention center was not built, their visit is a legitimate benefit.

In contrast, the rent revenues from local businesses using the convention center is a transfer. It benefits the city (and thus presumably taxpayers in general), but comes at the expense of the local businesses paying the rent. Note that no new cash has been generated here - it is simply a change in who holds the money.

The new catering jobs created are more difficult to assess. While it is true that these jobs are beneficial to those doing the work, if unemployment is not a chronic problem in Gallifrey, this represents an increase in the demand for workers. That is good for workers, as it increases wages, but hurts employers throughout the city who must now pay more to hire workers in a more competitive labor market. Thus, in most cases, these jobs will represent a transfer from local business to labor.
2. The appropriate shadow price here would represent the marginal cost of using additional resources to produce power at BPA. Since electricity is sold through regulated monopolies, it is unlikely that the regulated price represents this marginal cost. For example, recall from the lecture on monopolies that average cost pricing is often used to regulate natural monopolies, since marginal cost regulation could result in utilities not earning enough to cover large fixed costs.

The key to this answer is to think about the resources that a community has before and after the project - in this case, dam removal. After the dams are removed, BPA uses more resources to generate electricity. That is the cost to the community as a whole. While the market price represents the extra cost of electricity to the mill, the extra payments (above marginal cost) are not resources used up. They are a transfer of resources from the mill to the BPA.
3. a) In each case, we need to calculate the net present value of each project. For each discount rate, we will select the project with the highest net present value. We use the following formula to calculate the net present value. Note that costs or benefits that occur in year 0 are not discounted. Future benefits and costs are discounted as appropriate. In each case, we are given net benefits for a given year, and discount that value as appropriate. Thus:

$$
N P V=F V_{0}+\frac{F V_{1}}{(1+r)}+\frac{F V_{2}}{(1+r)^{2}}+\frac{F V_{3}}{(1+r)^{3}}
$$

where $F V_{t}$ is the future value of the net benefit in year $t$.
We begin by using the above formulas with a discount rate of $3 \%$ :

$$
\begin{gathered}
P V_{A}=-500+\frac{250}{(1.03)}+\frac{250}{(1.03)^{2}}+\frac{250}{(1.03)^{3}}=-500+242.71+235.65+228.79=\$ 207.15 \\
P V_{B}=50+\frac{50}{(1.03)}+\frac{50}{(1.03)^{2}}+\frac{50}{(1.03)^{3}}=50+48.54+47.13+45.76=\$ 191.43
\end{gathered}
$$

The net present value is higher for option A than for option B. Given this option $\mathbf{A}$ is preferable.
b) We repeat the calculations with a discount rate of 8\%:

$$
\begin{gathered}
P V_{A}=-500+\frac{250}{(1.08)}+\frac{250}{(1.08)^{2}}+\frac{250}{(1.08)^{3}}=-500+231.48+214.34+198.46=\$ 144.28 \\
P V_{B}=50+\frac{50}{(1.08)}+\frac{50}{(1.08)^{2}}+\frac{50}{(1.08)^{3}}=50+46.30+42.87+39.69=\$ 178.86
\end{gathered}
$$

The net present value is higher for option B than for option A. Given this option B is preferable.
c) A higher discount rate means that people place less importance on future outcomes. In option A, the costs are paid up-front, but the benefits come later. With a high discount rate (part b), these benefits are less important. In contrast, the future benefits receive more weight in part a.

Recall that the discount rate relates to interest rates. In part a, with a lower discount rate, the opportunity cost of having money now, rather than in the future, is lower. Thus, paying the up-front cost is not costly. In contrast, if alternative investments could earn an $8 \%$ return, paying the costs up-front, rather than investing them elsewhere, is costly.
4. a) To find the medicine sold when the price equals $\$ 25$ we plug the price into the demand curve:

$$
\begin{gathered}
25=50-0.0025 Q \\
0.0025 Q=25 \\
Q=10,000
\end{gathered}
$$



Note that we are given the price, not the marginal cost. Thus, you just need to plug the price into the demand curve. This is not a question about monopolies. You do not need to find marginal revenue, and even if you did, you would not equate marginal revenue to $\$ 25$, since that is not the marginal cost.

The willingness to pay for 10,000 bottles of medicine equals the total expenditure on medicine (area B) plus consumer surplus (area A). The value of each is:

$$
\begin{aligned}
& \text { area } A=\text { consumer surplus }=0.5(50-25)(10,000)=\$ 125,000 \\
& \text { area } B=\text { expenditure }=(25)(10,000)=\$ 250,000 \\
& \text { Willingness to Pay }=A+B=\$ 375,000
\end{aligned}
$$

b) We now need to find where the lower price of $\$ 1$ intersects demand:

$$
\begin{gathered}
1=50-0.0025 Q \\
0.0025 Q=49 \\
Q=19,600
\end{gathered}
$$



As before, the willingness to pay for 19,6 bottles of medicine equals the total expenditure on medicine (area B) plus consumer surplus (area $A$ ). The value of each is:

$$
\begin{aligned}
& \text { area } A=\text { consumer surplus }=0.5(50-1)(19,600)=\$ 480,200 \\
& \text { area } B=\text { expenditure }=(1)(19,600)=\$ 19,600 \\
& \text { Willingness to Pay }=A+B=\$ 499,800
\end{aligned}
$$

c) The benefit is the difference in willingness to pay when the price is $\$ 1$ compared to when the price is $\$ 25$. This is $\$ 499,800-\$ 375,000$, or $\$ 124,800$.
5. a) The cost of the project is a one-time construction cost. Thus, we do not need to discount this cost. However, the benefits, which are the cost savings to each middleincome family, occur annually. Thus, we need the present value of these benefits. Since the project goes on forever, we can use the rule of thumb for present value:

$$
P V=\frac{X}{r}
$$

In this case, the annual benefits are \$100,000 (= \$200 per year x 500 middle-income families). The discount rate is 0.05 .

$$
P V=\frac{100,000}{0.05}
$$

$$
P V=\$ 2,000,000
$$

The net present value equals the present value of the benefits minus the present value of the costs $(\$ 1,500,000)$, which equals $\$ \mathbf{5 0 0}, \mathbf{0 0 0}$. Since this is positive, the project is worth doing.
b) The only difference in part (b) is that the costs of construction are now financed by a $\$ 150$ per year tax on each middle-income family that owns an appliance. As a result, the net benefits to each family are just $\$ 50$ per year. For a single family, the net present value is:

$$
P V=\frac{50}{0.05}=\$ 1,000
$$

Since there are 500 middle income families, the net present value to the community as a whole is still $\$ 500,000$. Note that this is exactly the same as in part (a). The present value of the tax revenue is simply enough to cover the costs of construction. Thus, the project is still worth doing.

A common error here was to calculate the $\$ 500,000$ present value above, but then subtract the $\$ 1,500,000$ construction costs. Since the tax revenues are used to pay the construction costs, including the construction costs again double counts the costs of construction.
c) Here, we need to consider how each family is affected by the project. Since low-income families do not use electric appliances, they receive no benefits from the project. They do, however, now have a $\$ 100$ per year cost. Thus, the present value to each lowincome family is:

$$
\begin{array}{r}
P V=\frac{-100}{0.05} \\
P V=-\$ 2,000
\end{array}
$$

Middle-income families receive a net benefit of $\$ 100$ per year ( $=\$ 200$ energy cost savings minus the $\$ 100$ tax). The net present value for these families is thus:

$$
P V=\frac{100}{0.05}
$$

$$
P V=\$ 2,000
$$

Note that, since there are 500 middle-income families and 250 low-income families, the net present value of the project to the community as a whole remains the same as before:

$$
N P V=(\$ 2,000)(500)+(-\$ 2,000)(250)=\$ 500,000
$$

Thus, the only difference in part (c) is how the project is paid for. There is no right or wrong answer as to whether or not this changes your conclusion. Many people said that it did. They no longer supported the project when low-income families were harmed. However, some people still recommended the project, since it is good for the community as a whole.

