PAI 723 **Professor David Popp**
Solutions to Problem Set #4 **Fall 2024**

1. Begin with what we know. First, when there is 0 labor, total product must be equal to 0, which we see from the average product (AP = TP/L). When there is 1 unit of labor, total product is 180. Thus, average product equals 180, and marginal product = 180.

Now, proceed to 2 units of labor. The marginal product is 140. Thus, output increases by 140 units when a second worker is added. Total product thus equals 180 + 140, or 320. Average product equals 320/2, or 160.

With three workers, we are given total product, which equals 420. Average product equals 420/3, or 140. Marginal product is the change in output as labor increased from 2 to 3, which equals 100.

Finally, for 4 workers we are given average product. We can find total product by multiplying average product by the amount of labor. Thus, total product equals 120 x 4, or 480. To get marginal product, we need the change in total product when the fourth worker was added. This equals 60.

Labor	Total Product	Average Product	Marginal Product
0	0	**0**	**--**
1	**180**	180	180
2	320	160	**140**
3	420	140	100
4	**480**	**120**	60

(note: bold numbers are those given in the problem)

2. To answer this question, you simply need to compare the marginal product of adding each volunteer to one of the two neighborhoods. We want to place volunteers in the five most productive locations. These are the five options where marginal product is highest. This requires **three** volunteers Uptown and **two** volunteers Downtown.

We can see the intuition behind this by finding the best location for each of the five volunteers. For the first volunteer, the marginal product is 24 in Uptown, but only 16 Downtown. Thus, this worker should be assigned Uptown.

Now move to the second volunteer. The marginal product of a second worker Uptown is 20. This is still greater than the marginal product of 16 for a Downtown volunteer, so this second worker should also be assigned Uptown. However, the third volunteer will be more valuable Downtown, as the marginal product of the first volunteer there (16) is greater than the marginal product of a third worker Uptown (14).

Using similar logic, the fourth volunteer should also be assigned Downtown (MP of 14 vs. 13 for Uptown). The fifth volunteer should go Uptown, as the marginal product of 13 is better than the marginal product of 12 for a third worker Downtown.

Thus, the final allocation is **three** workers Uptown and **two** workers Downtown.

3. a) The mayor is confusing marginal cost and average cost. While it is true that the average cost of the 100 workers you know have is $4 per hour, new workers will need to be hired for the park project. Since no more federal funds are available, all of these workers will be hired at a cost of $7 per hour, for a total of $700.

b) Since the workforce is fixed at 100 workers, the cost of hiring them is now a sunk cost. They will be paid whether they work on the park project or a different project. Here, what matters is the opportunity cost of using these workers on the park, rather than elsewhere in the city. The cost of using these workers is the value of the project that they are taken off to fix up the park. Note that the city would presumably choose to pull workers off the least valuable projects. Still, the key question is whether that project is more or less valuable than fixing up the neighborhood park.

4. Because the volunteers do impose costs on the King Tutelage, we need to consider the productivity of a dollar spent on volunteers versus a dollar spent on professional staff. The marginal product per dollar spent on professional staff is 0.2 (= 5 students helped/$25). In contrast, volunteers only help 1 student per hour, and cost $10 per hour for training and supervision. Thus, the marginal product per dollar spent on volunteers is just 0.1 (=1/$10).

Based on these numbers, King Tutelage is not using enough professional staff. A dollar spent on professional staff currently helps twice as many students as a dollar spent on volunteers. They should hire more professional staff and fewer volunteers.

Note that we *cannot conclude* that they should only use professional staff. That is because the marginal products may change as the number of workers change. For examples, additional professionals might not be quite as productive, since presumably the most qualified workers were hired first. Similarly, the remaining volunteers may be more productive, and it may be easier to supervise a smaller number of volunteers. King Tutelage should re-evaluate their situation after hiring some more professional staff to see if the adjustments they have made are sufficient.

5. a) The marginal cost of earning a vote using leaflets is always $14. Thus, it makes sense to canvas in areas where the cost of earning a vote is less than $14 dollars. These neighborhoods are Apple Valle, Birch Hill, and Cottonwood. You can expect to earn 3 new votes from this effort.

b) The canvassing efforts from part (a) cost you $30 (= 7 + 10 + 13). This leaves you with $70 of the original $100 remaining, which you can spend on leaflets. Each new vote from leaflets costs $14. Thus, you will get 5 new votes from using leaflets (=70/14). In total, the $100 donation helps you get 8 new votes.