PAI 723 **Professor David Popp**
Solutions to Problem Set #3 **Fall 2024**

1. a) By looking at the graph, we can tell utility is not maximized because the indifference curve is not tangent to the budget constraint. Algebraically, we can tell utility is not maximized because marginal utility per dollar spent on each good (MU/P) is not equal.

b) Currently, the marginal utility per dollar spent on y is higher than for x. Intuitively, that means that we are getting more "bang for our buck" from consuming y, rather than x. As a result, we should consume less x, which will result in more y being consumed (since income is being held constant). For example, if we spend $1 less on x, utility will fall by $0.29. However, if we then spend $1 more on y, utility will increase by $0.87. Thus, the combined effect of spending $1 less on x and $1 more on y is that utility will increase by $0.58.

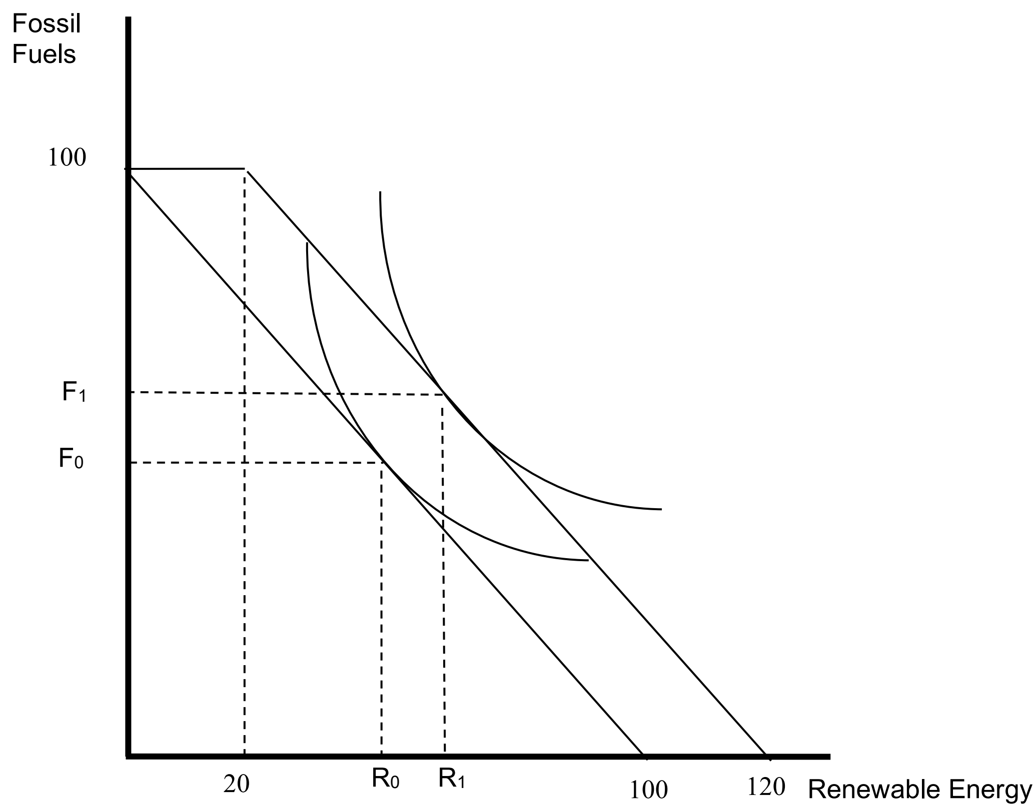
d) If you decreased consumption of x, the marginal utility per dollar spent on x should have increased. This occurs because of diminishing marginal utility. Since you now have less x, the remaining units are more valuable to you.

e) Similarly, as you decreased consumption of x, consumption of y increased. Thus, marginal utility per dollar of y is now lower. Since you have more, additional units of y are not as valuable to you as they were before.

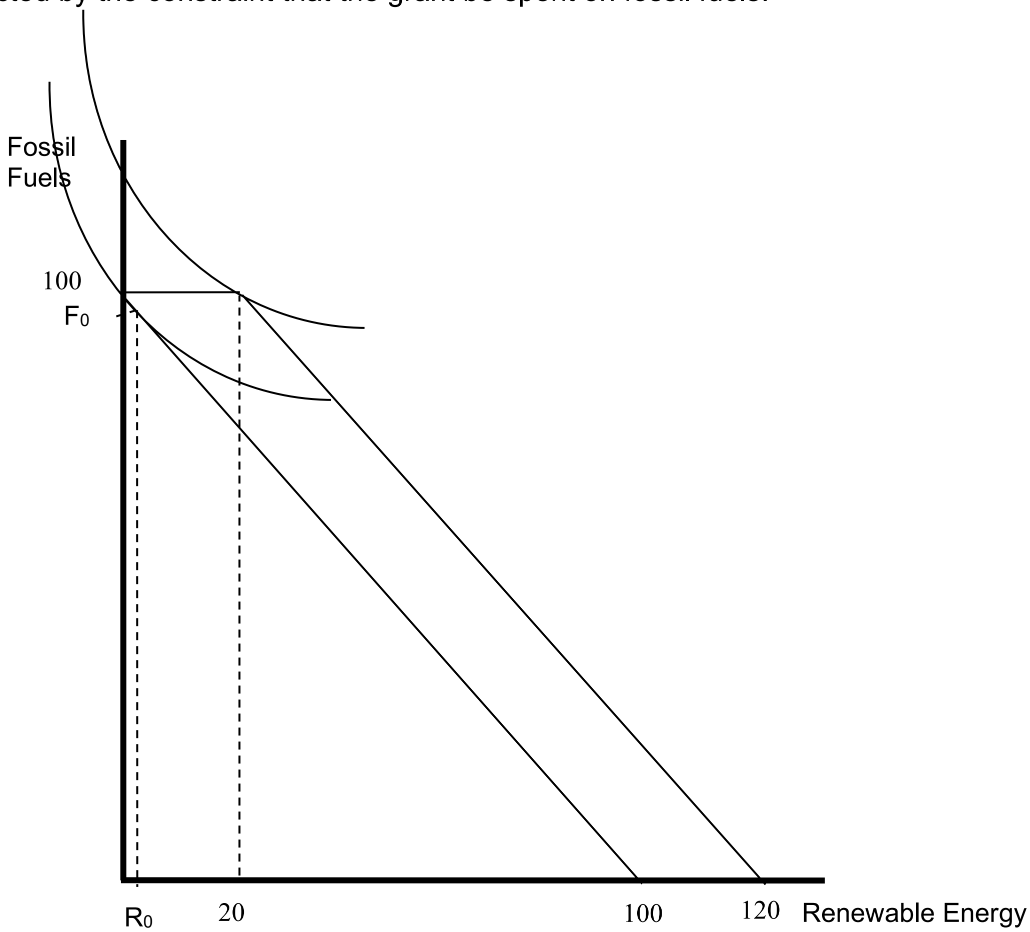
f) Utility is maximized when the marginal utilities per dollar are equal. At this point, it is no longer possible to swap consumption of one good for consumption of the other and still have utility increase. Graphically, the indifference curve is tangent to the budget constraint. Given the initial income and prices, utility is maximized when 10 of each good is consumed.

2. a) In this example, the country is already spending more than $20 million on renewable energy. Thus, the government can reallocate money it was previously spending so that they can spend more on both fossil fuels and renewable energy. The grant is equivalent to receiving $20 million of income.

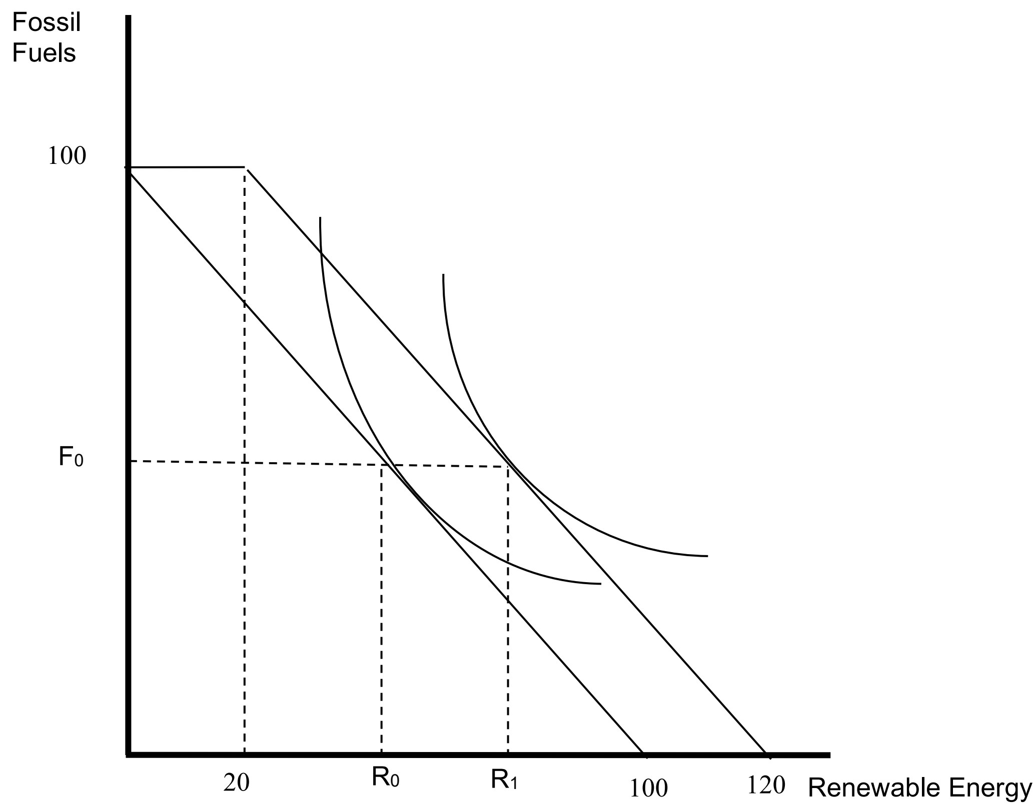
The *Economist* article that highlighted this dispute ("How green is my value?", October 21, 2017, p. 72-73.) even notes this possibility. The World Bank notes that funds from their grants go through national treasuries. Governments "can choose to finance fossil-fuel projects from general expenditures, not (the Bank's) cash specifically."



b) In this example, before receiving the grant, the country is spending most of its money on fossil fuels. Thus, it does not have much ability to reallocate money previously spent on renewables to fossil fuels. This is an example of a corner solution, where the country is affected by the constraint that the grant be spent on fossil fuels.



A common answer was to simply draw indifference curves in the middle of the diagram that only showed renewable energy increasing:

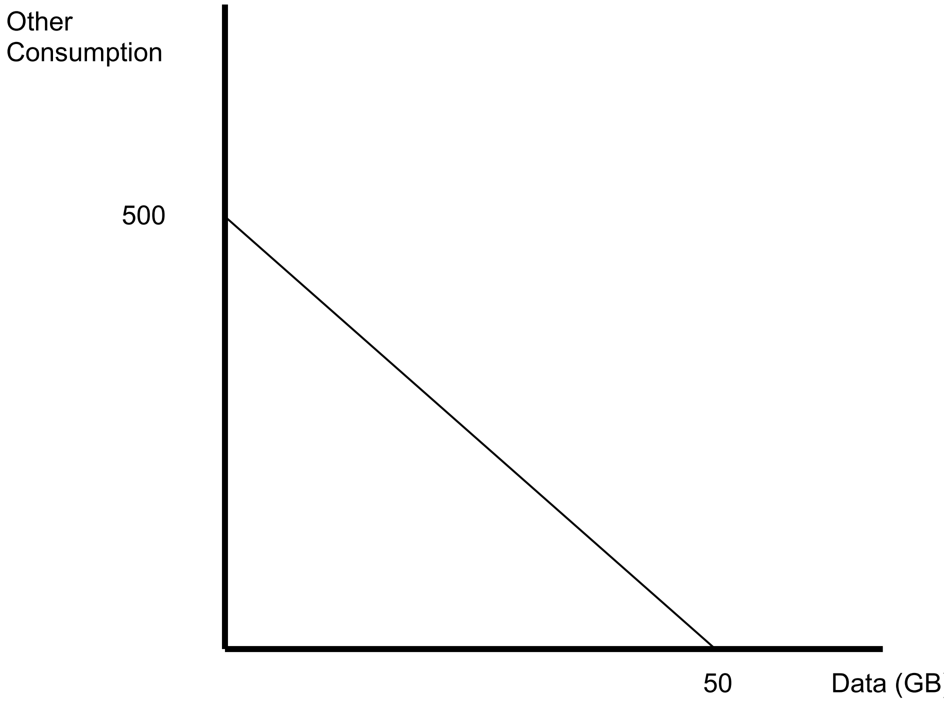


Since the graph is feasible (as long as you've drawn indifference curves that will not cross if extended further), I gave partial credit for this answer. However, I did not give full credit, as such a graph depends on an assumption that is unlikely to hold.

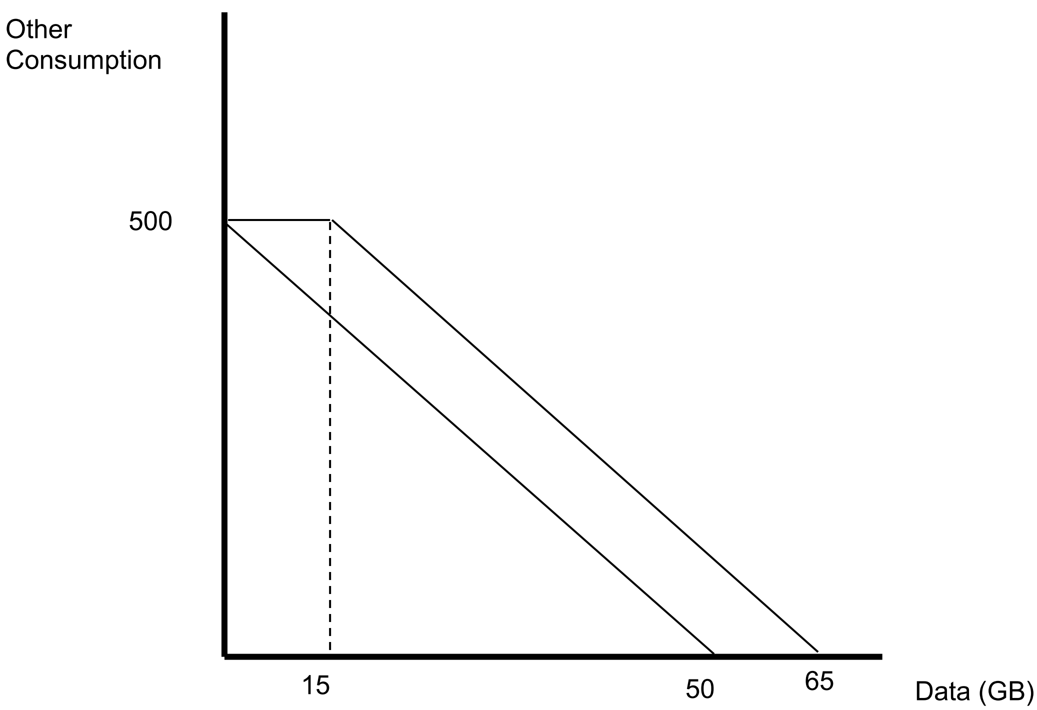
Recall that at the optimal point, $MU_F/P_F = MU_R/P_R$. This must hold at both tangencies above. Since the grant does not change the prices, the only way these can still be equal at the new tangency is if the marginal utility of renewables, MU_R, does not change as the country spends more on renewables – that is, if there are no diminishing returns to adding more renewables. Such an outcome is unlikely. For example, a country will build the first wind turbines in the windiest areas of the country, so that turbines built later will be less productive.

c) You need to know how much the recipient country is currently spending on renewable energy. If they are already spending more than $20 million on renewables, they will be able to reallocate some of that spending towards fossil fuels. On the graph, they are below the cutoff on the new budget constraint, so they are not affected by the constraint that the money must be spent on renewable energy.

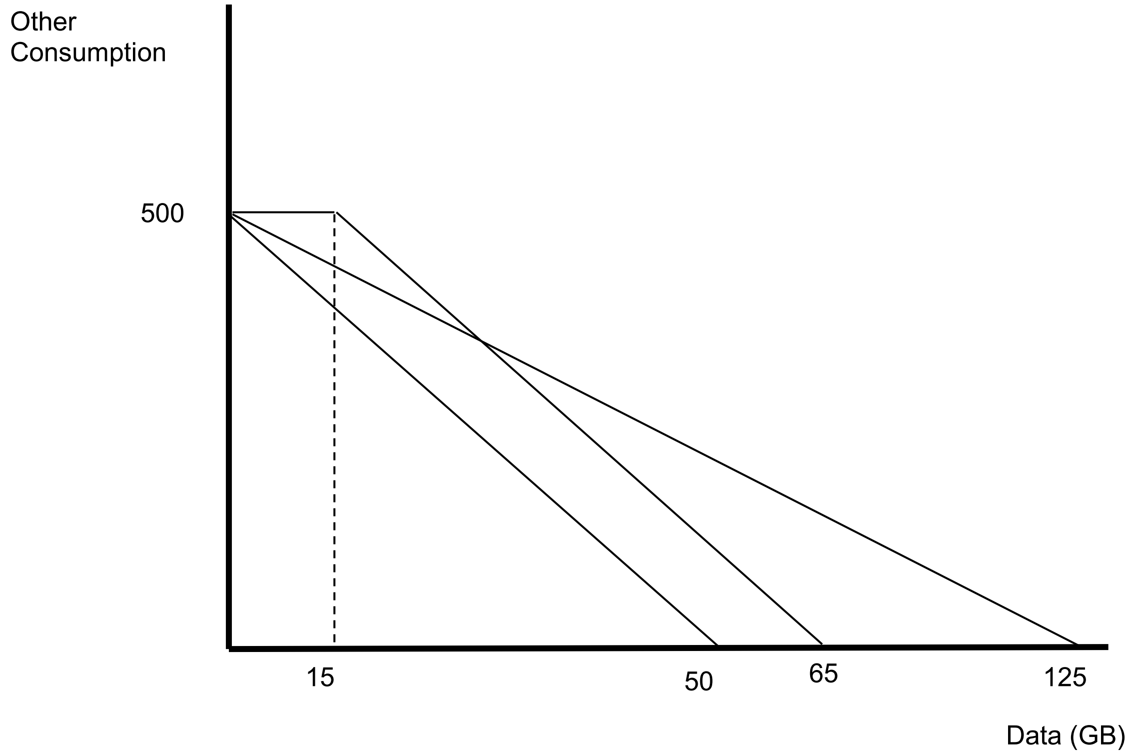
3. a) Without any aid, a typical low-income family can spend \$500 on other consumption, purchase 50 GB of data (= 500/10), or end up somewhere in between.



b) Now, this family can purchase up to 65 GB of data, and can purchase 15 GB of data even if they consume \$500 of other goods. The new budget constraint is a parallel shift out, but is cut off at \$500 of other consumption and 15 GB of data.

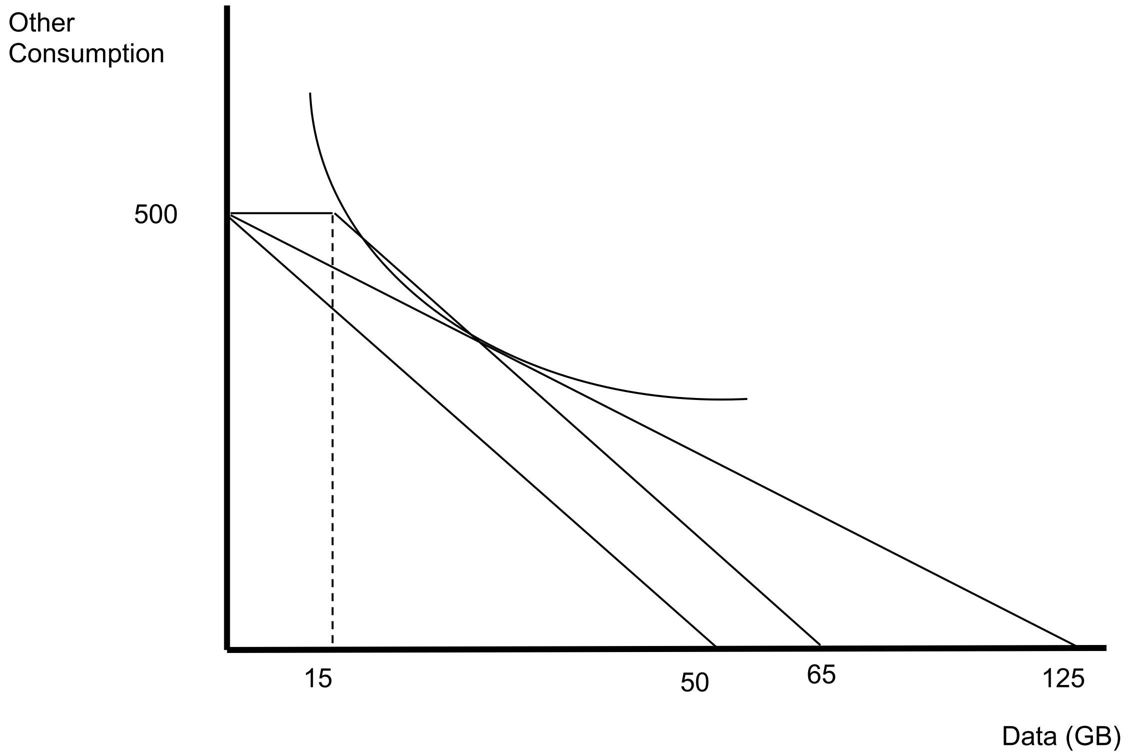


c) The subsidy lowers the price of data. Thus, the budget constraint rotates. If a family spends all their income on data, they now get 125 GB of data (= \$500/\$4).



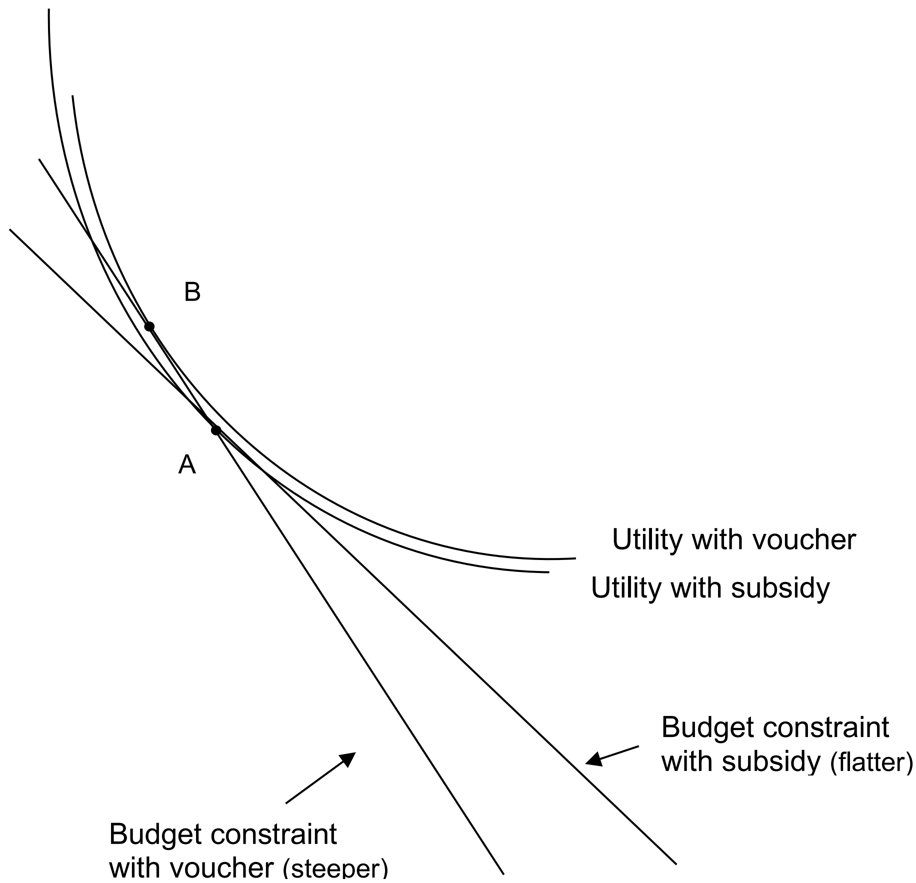
d) The key to this problem is that the budget constraints intersect at 25 GB of data. With the voucher, consumers get 15 GB of data free, and purchase an additional 10 GB of data for $10 each. Thus, they spend $100 to get 25 GB of data. Under the subsidy, the family can also purchase 25 GB of data for $100, since each GB of data only costs the consumer $4.

Given this, notice that if the family is maximizing utility under the subsidy at this intersection, then the budget constraint for the voucher program goes through their indifference curve.



Note that it is not correct to say that families are equally happy with either plan here. While it is true that it is *possible* for them to choose 25 GB of data under either plan, they won't want to do so. If their preferences are such that they choose 25 GB of data with the subsidy, they would prefer to have less data (but be on a higher indifference curve) if given the voucher instead.

We can see this better by enlarging the section of the graph where the lines intersect:

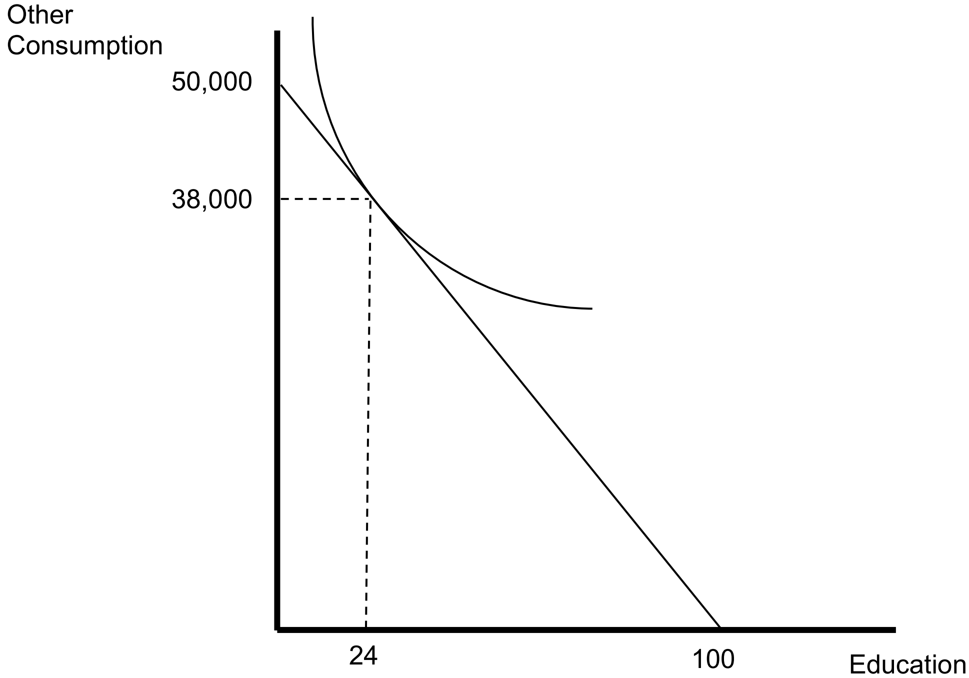


The budget constraint for the two policies intersect at 25 GB of data, which is also where the indifference curve is tangent to the subsidy budget constraint. Since the slope of the voucher budget constraint is steeper, the indifference curve cannot also be tangent to the voucher budget constraint there. Instead, *the budget constraint for the voucher plan goes through the indifference curve*. As a result, it is possible to draw a higher indifference curve that is tangent to the voucher budget constraint. Consumers are better off with vouchers.

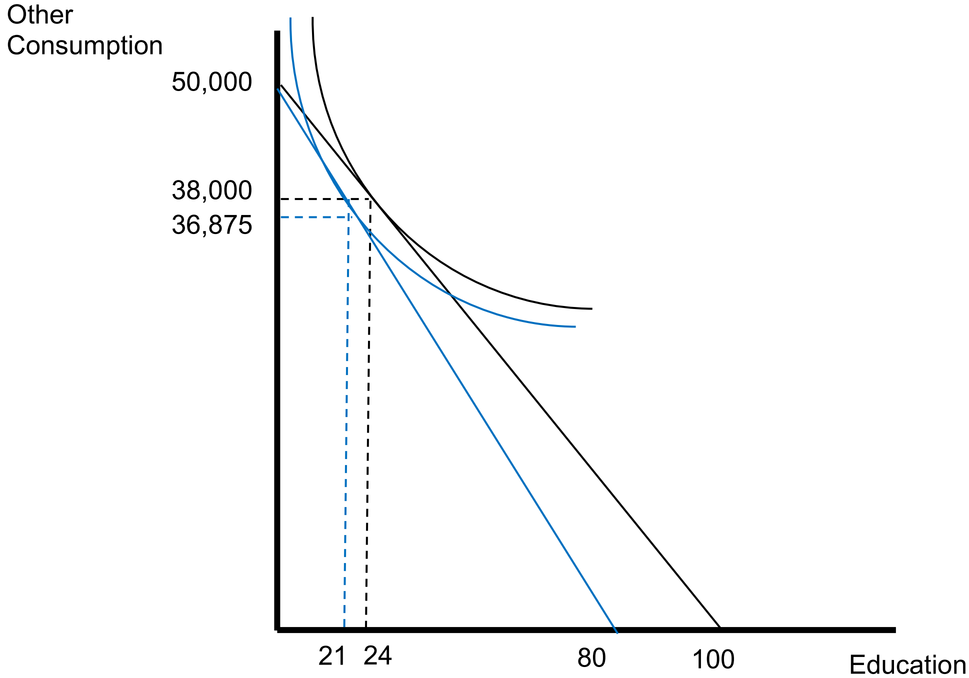
However, while consumers are happier with vouchers, they will consume less data than with the subsidy. Note that the new optimal bundle (point B under vouchers) is to the left of the optimal bundle under the subsidy (point A). The intuition is that the subsidy leads to greater consumption of data because it lowers the price of data. Thus, it induces families to substitute data for other goods. The key here is that, since the subsidy changes prices, it has both an income effect and a substitution effect. The voucher policy does not have a substitution effect, as the prices do not change.

Intuitively, since they intersect at point A, both plans give the consumer the same amount of money ($150). However, vouchers just have an income effect, as it gives a lump sum of money. The subsidy has both an income and substitution effect, since the value of money received depends on how much data a family buys. Thus, the subsidy encourages more data consumption. However, the family would rather receive the income with no strings attached, and thus prefer the voucher.

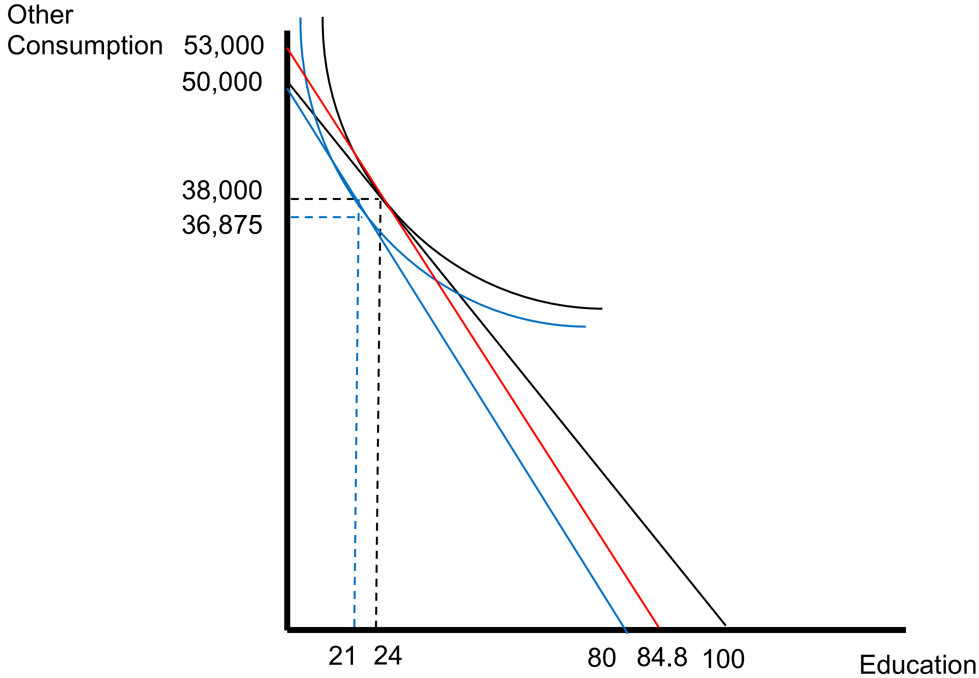
4. a) To draw the budget constraint, note that consumers can buy up to 100 credit hours of education (= $50,000/$500), or $50,000 worth of other goods. Note that these endpoints are what we need for the budget constraint – we want to show *what is possible*, not just what the consumers actually do. A typical family actually chooses 24 credit hours per year. Since each credit hour costs $500, this leaves them $38,000 to spend on other consumption, as 24 credit hours cost $12,000. This is shown by drawing an indifference curve tangent to the budget constraint at 24 credit hours of education. This is the highest possible indifference curve given the budget constraint.



b) The price increase rotates the budget constraint in, as shown in blue below. Now, the typical family chooses 21 credit hours, the family will have $36,875 left to purchase other goods (= $50,000 – 21(625)). The new indifference curve, tangent to the new budget constraint, is lower than the original indifference curve. Utility has fallen.



c) This budget constraint is shown in red below. The budget constraint is parallel to the blue constraint, since the price of tuition remains high, but shifts out by $3,000. Note that it goes through the original consumption point, since it is possible to consume both 24 hours of education and $38,000 of other consumption when total family income is $53,000. However, as we'll discuss further in part (d), the red constraint goes through the original indifference curve.



d) While it is possible for the family to purchase 24 credit hours, they will not. As noted above, the budget constraint goes through the original consumption point. However, given that education remains more expensive, they will substitute away from education. Thus, while they may spend some of the tax rebate on education, they will want to spend more of it on other consumption than they would have had tuition remained at $500 per credit hours.

The intuition on the previous page is all that was necessary to answer this question correctly. To see the response graphically, note that the red constraint must cross the original black constraint at the original bundle of 24 credit hours and $38,000 of other consumption. However, since the slope of the red line is steeper than the black budget constraint, it cannot also be tangent to the indifference curve at that point – only one of the budget constraints (the black one in this case) can be tangent, since the slopes must be equal for a tangency to occur. Thus, the red constraint must go through the original indifference curve. This is shown in the close-up of the intersection below. The family could choose a higher indifference curve with more spending on other consumption and less on education.