Problem Set #3
PAI 723
Professor David Popp
Fall 2024

Solutions Available Wednesday, October 23

1. This problem is designed to help you practice the concept of utility maximization. You will need to download the Excel worksheet utilmax.xls from the problems page of the class web site to complete the problem.

The worksheet includes a sheet labeled "calculations." On the calculations sheet, I have plotted an indifference curve (If you are interested, the curve is derived from the utility function U(x,y) =$\sqrt{xy}$, although you do not need to know that to complete the problem.)

Given the initial income and prices (I=\$20, P_x=\$1, P_y=\$1, X=15), the allocation of x and y is not optimal. Your task is to find the combination that does maximize utility. You may change any of the boxes that are highlighted on the calculations sheet (income, P_x, P_y, or X). You cannot change the consumption of y. This is found by spending all the remaining income. As you change variables, observe what happens to consumption of good y, marginal utility per dollar of each good, and total utility. Also, observe what happens to the indifference curve diagram. Included below are some questions to guide your use of this sheet.

a) Given the initial setup (I=\$20, Px=\$1, Py=\$1, X=15), utility is not maximized. How can we tell this by looking at marginal utility per dollar? How can we tell this by looking at the graph?
b) Given the relationship between the marginal utility per dollar of x and y, how should you change the level of x to improve utility? Why?
c) Try improving utility by changing x in the direction you state in question 2. Were you correct? If you were, total utility should increase. If you weren't, try moving x in the other direction (and be sure that you understand why you were wrong – if you don't, please ask me!)
d) What happened to the marginal utility per dollar of x as you went from (a) to (c)? Why?
e) What happened to the marginal utility per dollar of y as you went from (a) to (c)? Why?
f) Keep changing the value of x until a maximum is reached. How do you know when this has occurred?
g) (optional) Feel free to try different scenarios by changing the price or income, as well as the amount of x that is consumed.

2. The World Bank often provides financial support for renewable energy projects in developing countries. Recently, this funding has been criticized by environmentalists, who claim that increased funding for renewable energy has also allowed recipient countries to spend more on fossil fuel energy. They argue that for every additional dollar spent on renewable energy by recipient countries, spending on fossil fuel energy has increased by nearly as much. The World Bank disputes this claim, arguing that they have not provided funding for fossil fuel energy since 2010.

In the questions that follow, you are asked to draw budget constraints for a developing country before and after receiving funding from the World Bank. To begin, you may assume that the

country has a budget of $100 million to allocate on either renewable or fossil fuel energy sources.

a) Draw a hypothetical budget constraint for the country's energy spending before receiving World Bank funding, with millions of dollars spent on fossil fuels on the *y*-axis and millions of dollars spent on renewable energy on the *x*-axis. Suppose the World Bank offers a $20 million grant that can only be spent on renewable energy projects. Add a second budget constraint representing this grant. Add a set of indifference curves that are consistent with environmentalists' concerns that World Bank funding enables developing countries to spend more on fossil fuels.
b) Please reproduce your budget constraints from part (a). Add a set of indifference curves consistent with the World Bank's claim that their funding does not lead to more spending (or at least very little additional spending) on fossil fuels.
c) Based on your graphs, what would you need to know about a recipient country's current spending to say whether or not it was likely that the World Bank funding would enable them to spend significantly more on fossil fuels?

3. Arguing that access to the Internet is essential in the modern economy, Programmers Opposing Poverty (POP) advocates providing Internet services to low-income families. You have been asked to analyze two proposals designed to increase access by supporting data subscriptions on smartphones.

Proposal 1 would provide each family with vouchers that can be used to purchase 15 gigabytes (GB) of data each month. The vouchers can only be used to purchase data.

Proposal 2 would subsidize data. Currently, a gigabyte of data costs $10. For low-income families, the government would provide a subsidy, so that low-income families would only pay $4 per GB.

You have been asked to analyze the effect of each proposal on a typical low-income family with an income of $500 per month.

a) Draw a budget constraint for the typical family before aid is provided. For the *x*-axis, plot internet usage as gigabytes of data. Plot other consumption on the *y*-axis.
b) Reproduce your diagram from part (a). Now, add a budget constraint for proposal 1 (the voucher).
c) Reproduce your answer to (b). Then, add a budget constraint representing proposal 2 (the subsidy).
d) Your analysis shows that the typical family would buy exactly 25 GB of data when given the subsidy (proposal 2). Given this, under which proposal is the family better off? Which proposal encourages more data consumption? Explain. (*Hint*: How much of their own money must the family spend to get 25 GB of data under each of the two plans?)

4. Concerned about increasing tuition rates, Earnest Dummies Unite! (EDU!) proposes an income tax rebate designed to offset increased costs. This question asks you to evaluate their proposal. To begin, consider the following facts for a typical college-aged family:

- Before the increase, tuition rates are $500 per credit hour.
- The typical family with a college-aged student purchases 24 credit hours per year.
- The typical family has $50,000 of disposable income to spend on education or other consumption goods.

a) Draw a budget constraint and indifference curve for the typical family before rates increase. On the *x*-axis, education should be measured in credit hours purchased. On the *y*-axis, other consumption can be measured in dollars. Be sure to show the endpoints of the budget constraint, as well as the levels of credit hours and other consumption chosen by this family.

b) Reproduce your diagram from part (a). Now, consider an increase in tuition rates to $625 per credit hour. As a result of the rate increase, the typical family now only consumes 21 credit hours per year. Add the new budget constraint to the diagram, along with a new indifference curve showing the change in consumption patterns.

c) Reproduce your answer to (b). Now, consider the proposal from EDU!. They argue that families with college-aged children should receive a $3,000 tax rebate. This will allow them to afford the same combination of credit hours and other consumption that they had before tuition rates rose. Add a budget constraint representing this policy to your diagram.

d) Finally, consider the effects of the proposed tax rebate on welfare. When given the income tax rebate, will families choose to purchase 24 credit hours? How can you tell this? Compared to their utility in part (a) (before the tuition increase), are families worse off, better off, or the same when tuitions are higher and the tax rebate is in place? Explain intuitively why this is the case.